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DOPPLER SPECTRUM MATHEMATICAL MODEL OF SIGNAL SCATTERING FROM SEA SURFACE AT LOW GRAZING ANGLES

Abstract
Introduction. *Doppler spectra of signals scattered from the sea surface and received by radar are frequently used in oceanology and ecological monitoring applications. Existing Doppler spectrum models are limited in their application due to being based on empirical data obtained under changing conditions. The variability of observation conditions having a critical influence on microwave scattering by sea surface at low grazing angles is typical for marine radiolocations.*
Objective. *The aim of the investigation described in this article was to develop a mathematical model of Doppler spectra at low grazing angles for the microwave frequency range.*
Materials and methods. *The two-dimensional problem of the scattering of an electromagnetic field on a cylindrical deterministic surface is considered. The linear spatial sea spectrum model described by Elfohaily et al. is used to generate sea surface realisations. A solution to the scattering problem is obtained for the case of vertical polarisation of the incident electromagnetic field using an integral equation with the control of the error of the solution. A mathematical modelling of the Doppler Spectrum of signal scattered by sea surface is carried out using the statistical trial method. The case where the direction of the observation of the sea surface by radar is perpendicular to the direction of the wind is also considered. The electromagnetic field scattered in the direction of the radar receiver is calculated as a function of time for each generated sea surface realisation. Further, the set of variables of the implementation of scattered field is calculated for implementation of the Doppler spectrum.*
Results. *The set of implementations of the Doppler spectrum provided with its mathematical model consists of deterministic and random components. An approximation of each aforesaid component is proposed and mathematical expressions for the calculation of value components are presented. The modelling result is analysed.*
Conclusion. *The developed mathematical model is proposed for use in the design of an algorithm for sea surface condition estimation and pollutant detection using radar-detected signals.*

Key words: Radiolocation; Doppler spectrum of signal; modelling; radio wave scattering; sea surface; grazing angle of illumination

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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДОПЛЕРОВСКОГО СПЕКТРА СИГНАЛА, РАССЕЯННОГО МОРСКОЙ ПОВЕРХНОСТЬЮ, ПРИ СКОЛЬЗЯЩИХ УГЛАХ ОБЛУЧЕНИЯ

Аннотация

Введение. *Доплеровский спектр сигналов, рассеиваемых морской поверхностью и принимаемых радиолокатором, используется в различных задачах океанологии и экологического мониторинга. Существующие модели доплеровского спектра сигналов имеют ограниченное применение, поскольку получены на основе эмпирических данных в меняющихся условиях. Изменчивость условий наблюдения наиболее существенно влияет на рассеяние радиоволн на морской поверхности при характерном для морской радиолокации скользящем облучении.*

Цель исследования. *Разработка математической модели доплеровского спектра сигналов при скользящих углах облучения морской поверхности для сантиметрового диапазона длин волн.*

Материалы и методы. *Рассмотрена двумерная задача рассеяния электромагнитного поля на цилиндрической детерминированной поверхности. Для генерации реализаций морской поверхности использована линейная модель с пространственным спектром морского волнения Эльфохейли. Получено решение задачи рассеяния для случая вертикальной поляризации падающего электромагнитного поля методом интегрального уравнения с контролем погрешности расчета. Методом статистических испытаний проведено математическое моделирование доплеровского спектра сигналов, рассеиваемых морской поверхностью. Рассмотрен случай, когда направление облучения морской поверхности радиолокатором перпендикулярно направлению ветра. Для каждой из сгенерированных реализаций морской поверхности рассчитано электромагнитное поле, рассеиваемое в направлении на приемник радиолокатора, как функция времени. Далее по совокупности временных реализаций рассеянного поля вычислена реализация доплеровского спектра сигналов.*

Результаты. *По совокупности реализаций доплеровского спектра получена его математическая модель, содержащая детерминированную и случайную составляющие. Предложена аппроксимация каждой из указанных составляющих; приведены математические выражения для их расчета. Приведен анализ результатов моделирования.*

Заключение. *Полученную математическую модель доплеровского спектра предположено использовать для разработки алгоритмов оценки по принятым радиолокационным сигналам состояния морской поверхности и наличия на ней загрязняющих веществ.*

Ключевые слова: радиолокация; доплеровский спектр сигнала; моделирование; рассеяние радиоволн; морская поверхность; скользящий угол облучения

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Introduction. Microwave radar stations are widely used for the remote sensing of a sea surface [1]–[4]. Among the advantages of remote ocean probing carried out by radar are its all-weather capability, independence of day- or night-time operation and possibility of radar installation on stationary objects as well as movable carriers. The abovementioned advantages of radar provide the possibility to obtain information about a sea surface in a short time period, which is highly important for an operational analysis of an ecological situation in investigating water aria.

Doppler spectra of signals (DSS) received by a radar are used for determining the investigated characteristics of sea surface (roughness, velocity and direction of near-water wind) and also for ecological monitoring of the sea surface in the presence of biological and non-biological (oil films) pollutants [1]–[5].

Radar signal reflection from the sea surface is affected by the velocity and direction of a near-surface wind as well as its duration, the length of the wind acceleration region, the presence of pollution (e.g. oil films) and other local weather conditions [6], [7]. In addition, there are complications affecting the calibration of sea surface reflected signals [6].

The theoretical framework for the scattering of radio waves by a rough sea surface became the basis for the interpretation of experimental results. Within the this framework are included both resonant (Bragg) and non-resonant scattering [2]–[7].

Current empirical microwave mathematical models are represented as DSS formulae averaged on a number of realisations for both downwind and upwind illumination directions [5]. However, researchers have yet to derive any DSS mathematical models for illumination directed perpendicular to the wind direction for the wavelength of interest. In [4], only particular DSS realisations for the wind-perpendicular illumination direction are presented.

In order to elaborate algorithms for the inverse problem of reconstructing sea surface characteristics using received radar signals deflected at low grazing angles, averaged data from variety of DSS realisations is not sufficient. In particular, such an algorithm requires a probability model, which allows DSS realisations to be generated in accordance with given parameters and the accuracy of solution to the scattering problem to be estimated.

The development of a theoretical approach to sea surface diffraction of radio waves at low grazing angles allows the creation of new mathematical DSS models for surface areas, whose size is determined by spatial radar resolution, along with various wind velocity and direction parameters.

The aim of this paper to develop a mathematical DSS model at low grazing angles for the microwave frequency range typical of radars utilised for oceanology and ecological monitoring tasks.

In order to develop the described mathematical model, it was deemed necessary to carry out the following steps:

– select the mathematical model of the rough sea surface;

– choose a method for solving the problem of the scattering of radio waves by generated sea surface realisations;

– develop the mathematical model for the scattering of a microwaves at low grazing angles using the statistical trial method;

– process the modelling results and form the mathematical model of DSS scattered by the sea surface;

The mathematical DSS model, which has a practical application and is hitherto poorly investigated in scientific literature, will be formed for cases when the sea surface radar illumination direction is perpendicular to the direction of the wind.

Sea surface model. There are variety of models for describing the sea surface, including linear and nonlinear models that take into account the spatial spectra of sea waves [10]–[13].

Within the framework of the linear model, the sea surface is represented as a sum of spatial harmonics whose amplitudes are comprised of independent Gauss random values with a dispersion depending on the wavenumber of a radial sea wave spectrum. A review of common nonlinear sea surface models is represented in [9]–[11].

The Elfohaily spectrum, which takes the contribution of capillary and gravity-capillary waves into account, is adopted as a spatial spectrum of sea roughness [12], [13].

Since a description of sea wave breaking and foam formation processes are rather complicated, we will rely on a linear, one-dimensional sea surface model due to its simplicity in realisation and correctness for low wind speeds. Mathematical expressions sufficient for the generation of sea surface realisations can be found in [9]–[11].

Solving the scattering problem. In order to form the mathematical DSS model from a rough sea surface at low grazing angles, it is necessary to obtain information concerning the field scattered toward a radar by solving the problem of the diffraction of radio waves on the sea surface.

Previously, the so-called two-scale model, within the frames of which the scattered field consists of two components, was used to obtain the signal characteristics [14]–[16]. The first component, which corresponds to scattering from a significant roughness of a sea surface (gravity waves), is defined by the Kirchhoff method. The second component, which can be defined by the perturbation method, corresponds to scattering from small-scale roughness (ripples). However, the results obtained using two-scale model at low grazing angles differ from those obtained experimentally. Moreover, the obtained results were not evaluated in terms of their accuracy.

The integral equation method (IEM), which is an effective tool for the theoretical investigation of diffraction problems, as well as having validity for obtaining numerical algorithms for solving a broad class of similar problems, is widely used for overcoming of mentioned difficulties due to intense development of a computer hardware [13]–[15]. In addition, the above-mentioned method comprises a rigorous numerical method for solving the diffraction problem obtaining solutions that fulfil the Maxwell equations. IEM is also suitable for use at low grazing angles. Within the framework of this method, the following integral equations are distinguished [17]–[19]:

$$\mathbf{E}_{\text{inc}}(r) = \int_S \frac{\partial \mathbf{E}(r')}{\partial n'} G(r, r') dS'; \tag{1}$$

$$\mathbf{H}(r) = 2\mathbf{H}_{\text{inc}}(r) + 2\int_S \mathbf{H}(r') \frac{\partial G(r, r')}{\partial n'} dS', \tag{2}$$

where $\mathbf{E}_{\text{inc}}(r)$, $\mathbf{H}_{\text{inc}}(r)$ are the intensities of incident electric and magnetic fields, respectively; $\mathbf{E}(r)$, $\mathbf{H}(r)$ are the intensities of total electric and magnetic fields on surface *S*, respectively; $G(r, r')$ is the Green function; n' is the external normal of surface *S*; *r* is the point of view; r' is the integration point.

Let's consider the two-dimensional problem of electromagnetic field scattering from a deterministic rough cylinder surface *S*. It is typical of marine radar at grazing angles that the longitudinal dimension of the illuminated area of a sea surface, which is sufficient for radio waves scattering, far exceeds the transverse dimension. Therefore, a two-dimensional approximation of the solution is adequate.

Surface roughness types and geometrical properties characterising the problem are shown in Figure 1. The source and receiver of electromagnetic waves are positioned at point *A*. The incident electromagnetic wave is characterised by electric field intensity $\mathbf{E}_{\text{inc}}$, magnetic field $\mathbf{H}_{\text{inc}}$, and wave vector $\mathbf{K}_{\text{inc}}$; the scattered wave is characterised by the intensity of electric field $\mathbf{E}_{\text{sc}}$, magnetic field $\mathbf{H}_{\text{sc}}$, and wave vector $\mathbf{K}_{\text{sc}}$. The OZ-axis of the coordinate system is perpendicular to the incidence plane of the electromagnetic wave. Sea surface is illuminated at angle θ_{inc}, while the scattering angle of electromagnetic energy is θ_{sc} *.

A vertical incident field polarisation was chosen since the performance of radar having this type of polarisation provides a higher radar sea surface cross-section value at low grazing angles compared with horizontal polarisation. Moreover, this type of polarisation is used more frequently in marine radars used to carry out oceanic investigations [16].

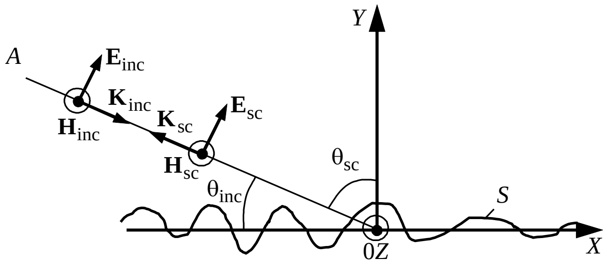

Fig. 1. Geometrical characteristics of a rough surface

The integral equation should be transformed into system of linear equations (SLQ) having unknown variables, which comprise the expansion coefficients of the solution having the selected basis function [17]–[18].

The generated sea surface realisation is divided into *N* plots. A step function is used to represent the desired solution (surface current density) inside each plot. The utilisation of this function allows the simplest numerical algorithm to be obtained [17]–[18].

In the present paper, the integral equation (2) is used for solving the scattering problem; an analogous solution could be obtained using equation (1).

Let us assume that the result of SLQ solutions is represented by *N* unknown coefficients of desired solution expansion. We can rewrite SLQ in matrix form [17]–[19]:

$$A\mathbf{J} = \mathbf{H}_{1\text{inc}}, \tag{3}$$

where *A* is the *N*-by-*N* impedance matrix; **J** is the column-vector of the surface current density consisting of *N* elements; $\mathbf{H}_{1\,\text{inc}}$ is the column-vector of incident field magnitude consisting of *N* elements.

The elements of the impedance matrix for vertical polarisation were calculated according to the formulae [17], [20]:

$$A(m, n) = \begin{cases} -\dfrac{ik\Delta x}{4} H_1^{(1)}(kR_{mn}) \times \\ \quad \times \left[\dfrac{y(x_m) - y(x_n) - y'(x_n)(x_m - x_n)}{R_{mn}} \right], \quad m \neq n; \\ \dfrac{1}{2} - \dfrac{\Delta x y''(x_m)}{4\pi l_y(x_m)} + \\ \quad + \dfrac{k\Delta x l_y(x_m)}{4} \left[1 + \dfrac{2i}{\pi} \ln\left(\dfrac{k\Delta x l_y(x_m)}{4} \right) \right], \quad m = n, \end{cases} \tag{4}$$

* It is customary to measure the illumination angle from the un-roughed sea surface and the scattering angle from the normal surface.

where m is the point of view index, n is the integration point index $\left(m,\ n=\overline{1,\ N}\right)$*; i is the imaginary unit; k is the wave number; Δx is the single-plot sea surface length; $H_1^{(1)}$ is the first-kind, first-order Hankel function;

$$R_{mn}=\sqrt{\left[y\left(x_m\right)-y\left(x_n\right)\right]^2+\left(x_m-x_n\right)^2};\qquad(5)$$

y', y'' are the first and the second derivatives of the x-coordinate with respect to the y-coordinate, respectively;

$$l_y\left(x_m\right)=\sqrt{1+\left[y'\left(x_m\right)\right]^2}.\qquad(6)$$

Virtual resistive insets representing plane surface sections of given length L_r with variable resistance $R_0(x)$ are placed at the boundaries of the considered surface plots [19].

$$R_0(x)=\begin{cases}0,\ |x|<L_t/2;\\ Z_0\left(\dfrac{0.5L-|x|}{L_r}\right)^4,\ L_t/2<|x|\le L_t/2+L_r,\end{cases}\qquad(7)$$

where $Z_0=120\pi$ is the wave impedance; L_t is the length of the generated sea surface plot; $L=L_t+L_r$ is the combined length of the sea surface plot; L_r is the length of the resistive inset.

Diagonal elements of impedance matrix A corresponding to the vertically-polarised intrinsic field are calculated using formulae, which takes into account the resistive inset [19]–[20]:

$$A(m,\ m)=R_0\left(x_m\right)+\frac{1}{2}-\frac{\Delta x y''\left(x_m\right)}{4\pi l_y}+$$
$$+\frac{k\Delta x l_y\left(x_m\right)}{4}\left[1+\frac{2i}{\pi}\ln\left(\frac{k\Delta x l_y\left(x_m\right)}{4}\right)\right].\qquad(8)$$

The straightforward calculation of the far zone components of the surface-scattered electromagnetic field was carried out using the well-known formulae [18], [21].

The joint tolerance of all calculations during obtaining of the sea surface field was estimated using the energy conservation law. According to this law, the power of incident field P_{inc} must be equal to the power of scattered field P_{sc} with respect to the part of power absorbed by the resistive inset P_{ins}:

$$P_{\text{inc}}=P_{\text{sc}}+P_{\text{ins}}.\qquad(9)$$

The power of incident field was calculated using the formula [18], [21]:

$$P_{\text{inc}}=Z_0H_0^2\sin\theta_{\text{inc}},\qquad(10)$$

where H_0 is the amplitude of incident field intensity.

The power of the field scattered by the sea surface was calculated by the formula [21]:

$$P_{\text{sc}}=\frac{k}{8\pi}\frac{Z_0}{2}\int_{-\pi/2}^{+\pi/2}\left|W\left(\theta_{\text{sc}}\right)\right|^2d\theta_{\text{sc}},\qquad(11)$$

where

$$W\left(\theta_{\text{sc}}\right)=\int_{-L/2}^{+L/2}J(x)ik\left[-y'(x)\sin\theta_{\text{sc}}+\cos\theta_{\text{sc}}\right]\times$$
$$\times\cdot\exp\left\{ik\left[-x\sin\theta_{\text{sc}}+y(x)\cos\theta_{\text{sc}}\right]\right\}dx,\qquad(12)$$

and $J(x)$ is the surface current density calculated according to (3).

The power absorbed by resistive insets was calculated by formula:

$$P_{\text{ins}}=H_0^2\sin\theta_{\text{inc}}\int_{-L/2}^{+L/2}R_0(x)dx.\qquad(13)$$

Finally, joint tolerance of all calculations during scattering was estimated by [21]:

$$\delta=1-\left(\frac{P_{\text{sc}}+P_{\text{ins}}}{P_{\text{inc}}}\right).\qquad(14)$$

Mathematical modelling of DSS. Modelling of the DSS was carried out in three steps using the MATLAB software.

First, time realisations of the sea surface plots of given length (520 realisations in total separated by the time interval Δt) for the constant value of root-mean-square (RMS) deviation of the sea surface y-coordinate σ_y were generated.

* Point of view is a point on the sea surface where the surface current density is calculated. Point of integration is a point on the sea surface whose electromagnetic field contributes to surface current density at the point of view.

Table 1. Parameters for generating of the sea surface model

Parameter	Value
Wavelength of incident field, m	0.03
Length of sea surface plot, m	10
Sea surface illumination angle, θ_s, ...°	2
Length of single section of sea surface Δx, m	0.01
Root-mean-square (RMS) deviation of sea surface *y*-coordinate, σ_y, m	0.025; 0.1
Time interval betweensea surface realisations, Δt, s	0.0135
Duration of analysis interval, T_a, s	7

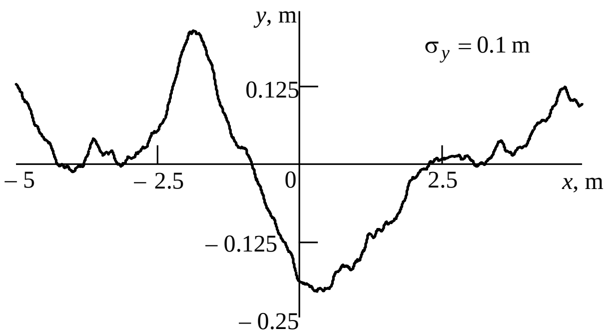

Fig. 2. An example of the implementation of the sea surface plot

The DSS modelling was carried out for two values of the root-mean-square deviation of the sea surface *y*-coordinate for comparison and further analysis. Table 1 lists the parameters used for the generation. Fig. 2* depicts an example of the realisation procedure.

Second, the time dependencies of electromagnetic field scattered toward the radar for all sea surface realisations was calculated in accordance to the formulae (2)–(14) using IEM. Herein, the wind direction determining sea wave propagation was assumed to be perpendicular to the sea surface illumination direction. The error of the scattering problem solution did not exceed 25% on average.

During the third step of modelling, the dependence of the DSS on power at the sea surface was carried out in according to the formula [11]:

$$S(f_D, \theta_{inc}, \theta_{sc}) = \frac{1}{T_a}\left|\int_0^{T_a} u(t, \theta_{inc}, \theta_{sc})\exp(-j2\pi f_D t)\,dt\right|^2,$$

where T_a is the analysis interval duration, *u* is the field scattered by the sea surface in the radar receiving point; f_D is the Doppler frequency shift; *t* is time.

The 7 s duration of the signal analysis interval for the DSS calculation was sufficient to achieve the necessary resolution of the Doppler frequency. The operations were repeated 100 times according to the described steps in order to obtain the necessary quantity of DSS realisations. Then the obtained data was processed to form the DSS numerical model and to make further analysis.

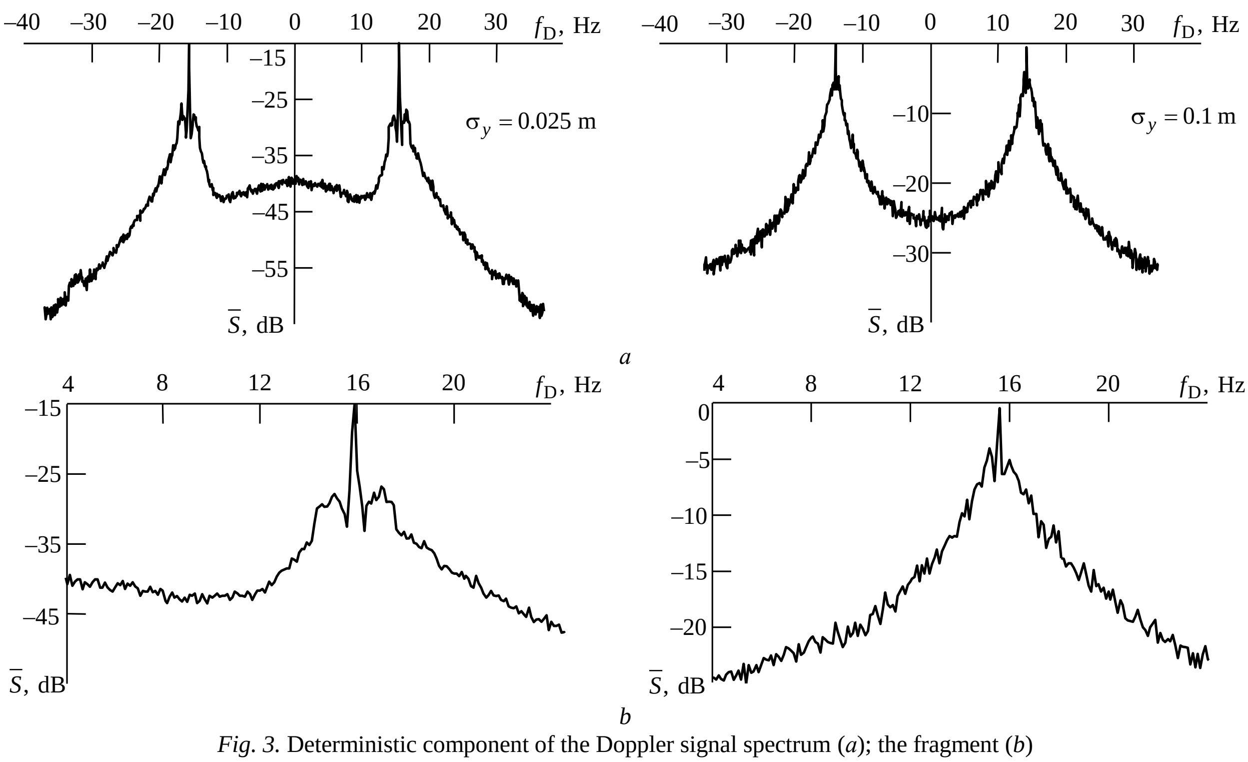

Fig. 3. Deterministic component of the Doppler signal spectrum (*a*); the fragment (*b*)

* The origin of coordinate system was placed in the middle of illuminated section of unroughed sea surface.

Table 2. Parameters of the characteristic elements for the region of the positive frequencies of the Doppler spectr

DSS parameter	σ_y, m	
	0.025	0.1
Main maximum frequency f_{max}, Hz	15.6	15.6
Width of the main maximum, Hz	0.40	0.45
Relative amplitude of the main maximum, dB	–19.2	–4.27
Frequency of the left additional maximum, Hz	14.6	15.0
Frequency of the right additional maximum, Hz	16.6	16.2
Relative amplitude of the left additional maximum, dB	–28.6	–10.4
Relative amplitude of the right additional maximum, dB	–27.6	–10.5

Calculations were carried out using a personal computer running Windows 7 OS having an Intel Core i5 2430M (2×2.4 GHz) processor and 4 GB of onboard RAM. On average, the DSS calculation of the single sea surface realisation took 12 s. The entire modelling and DSS calculation process took eight days.

During further analysis DSS was considered as an additive mixture of deterministic $\overline{S(f_D)}$ and random ΔS components:

$$S(f_D) = \overline{S(f_D)} + \Delta S.$$

Fig. 3, *a* demonstrates deterministic component of DSS for different RMS deviations of the sea surface *y*-coordinates. Fig. 3, *b* shows scaled-up fragments of these DSS.

In analysing Fig. 3, *a*, attention should be paid to two main maxima or so called “Bragg lines” caused by resonant scattering of radio waves incident onto the sea surface and corresponding to incoming and outgoing sea waves. Two additional peaks should also be noted in the area of the main peak; these are caused by the resonant and nonresonant scattering mechanism.

Next, in order to simplify subsequent analysis, we will consider only the part of DSS that corresponds to positive frequencies (Fig. 3, *b*); the negative frequency analysis method is the same. The Bragg line frequency positions, as well as width and other parameters are displayed in the Table 2.

The frequency position of the main maximum of the deterministic DSS component is calculated by formula [2]:

$$f_{D\,max} = \sqrt{gK_w + (\sigma_w/\rho_w)K_w^3}\Big/2\pi, \qquad (15)$$

where $g = 9.81\ \mathrm{m/s^2}$ is the gravity constant; $K_w = 2\pi/\Lambda_w$ is the wave number of the sea wave; $\sigma_w = 74.3 \cdot 10^{-3}$ N/m is the surface-tension on the air-sea interface; $\rho_w = 10^3\ \mathrm{kg/m^3}$ is the density of the sea water, and

$$\Lambda_w = \lambda/(2\cos\theta_{inc}) \qquad (16)$$

is the length of sea wave that produces the main maximum of the deterministic DSS component; λ is the length of the incident radio wave.

Calculations carried out using the formulae (15) and (16) are used to fix the frequency values of the main peak of the deterministic DSS component (see Table 2).

Considering the character of the deterministic DSS components as a frequency function (one main maximum, two additional maxima and two sections along the edges), it is proposed to split them into five nonoverlapping frequency intervals and carry out independent approximations of each of them:

$$\bar{S}(f_D) = \sum_{m=1}^{5} F_m(f_D),$$

where

$$\begin{cases} F_m(f) = a_{P,\,m} f^P + a_{P-1,\,m} f^{P-1} + \mathrm{K} + a_{0,\,m}, \\ \qquad\qquad f \ge f_{1,\,m},\ f \le f_{2,\,m}; \\ F_m(f) = 0,\ f < f_{1,\,m},\ f > f_{2,\,m}. \end{cases}$$

Table 3. Simulation options for $\sigma_y = 0.025$ m

Parameter	$\sigma_y = 0.025$ m				
	m				
	1	2	3	4	5
P	4	3	3	3	4
$f_{1,\,m}$, Hz	0	13.9	15.3	16	17.3
$f_{2,\,m}$, Hz	13.9	15.3	16	17.3	37
$a_{4,\,m}$	$2.7 \cdot 10^{-3}$	0	0	0	$-4.2 \cdot 10^{-5}$
$a_{3,\,m}$	$-55.2 \cdot 10^{-3}$	2.753	226.75	1.045	$2.4 \cdot 10^{-3}$
$a_{2,\,m}$	0.336	–129.22	$-1.08 \cdot 10^4$	–62.6	0.041
$a_{1,\,m}$	–0.921	$2.01 \cdot 10^3$	$1.7 \cdot 10^5$	$1.22 \cdot 10^3$	–5.504
$a_{0,\,m}$	–39.386	$-1.04 \cdot 10^4$	$-8.95 \cdot 10^5$	$-7.73 \cdot 10^3$	41.16

Table 4. Simulation options for $\sigma_y = 0.1$ m

Parameter	$\sigma_y = 0.1$ m				
	m				
	1	2	3	4	5
P	4	3	3	3	4
$f_{1,m}$, Hz	0	14.3	15.3	16	17.3
$f_{2,m}$, Hz	14.3	15.3	16	17.3	37
$a_{4,m}$	$8.79 \cdot 10^{-4}$	0	0	0	$-3.9 \cdot 10^{-4}$
$a_{3,m}$	$-17 \cdot 10^{-3}$	–1.04	236.84	5.467	$36.7 \cdot 10^{-3}$
$a_{2,m}$	0.19	$1.37 \cdot 10^{3}$	$1.11 \cdot 10^{4}$	–276.45	–1.16
$a_{1,m}$	–0.634	$-2.02 \cdot 10^{4}$	$1.76 \cdot 10^{5}$	$4.65 \cdot 10^{3}$	12.1
$a_{0,m}$	–31.22	$9.92 \cdot 10^{4}$	$-9.23 \cdot 10^{5}$	$-2.61 \cdot 10^{4}$	–33.85

The values of $a_{P,m}$, $f_{1,m}$, $f_{2,m}$, P are shown in Tables 3 and 4.

The average error of deterministic DSS component approximation was 0.13% for RMS deviation of the sea surface, the *y*-coordinate is equal to 0.025 m and 0.17% for RMS deviation equals 0.1 m.

The random DSS component was determined using the statistic model, taking the form of a random stationary process with zero mathematical expectation and given correlation function (CF) and RMS deviation. By analysing the modelling data using the Pearson criterion, it was shown that random component of DSS is equal to 5.6 dB for $\sigma_y = 0.025$ m and 6.05 dB for $\sigma_y = 0.1$ m.

The normalised CF of the random DSS component $R_n(\Delta f_D)$ is shown in Fig. 4. The correlation interval of the random DSS component for $\sigma_y = 0.025$ m was equal to 0.59 Hz, while for $\sigma_y = 0.1$ m it was equal to 3.85 Hz. Correlation interval values are expressed in frequency units because calculated value, the random DSS component depends on it.

Conclusion. The mathematical model of DSS of microwave range radio waves with vertical polarisation including deterministic and random DSS components for the illumination direction perpendicular to wind action was elaborated. Mathematical expressions for these components for two different RMS deviations of sea surface *y*-coordinates which corresponding to low sea force is obtained.

The following conclusions could be made during analysis of obtained results for fixed grazing angle and increasing RMS deviation of the sea surface *y*-coordinate:

1. The frequency position of the main maximum stays constant. This conclusion is approved by the theoretical calculations.

2. Width of the main maximum of deterministic DSS component (on –3 dB level), which includes information about the wind influence on the sea surface increases. Increase of the wind speed above the sea surface leads to increase of the RMS deviation of the sea surface *y*-components.

3. Relative amplitude of main and additional maximums of the deterministic DSS components increases. The sea surface becomes rougher and the coefficients of radar directed backscattering increases.

4. Difference of relative amplitudes of main and additional DSS maximums decreases. This effect is caused by an increase in the quantity of the sea surface plots that perform nonresonant scattering due to

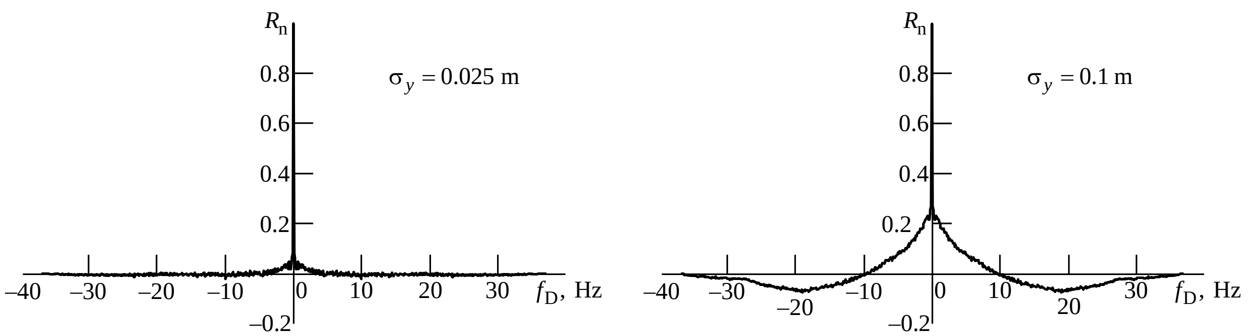

Fig. 4. The normalized correlation function of the random component of the Doppler signal spectrum

an increase in sea roughness.

Therefore, the developed mathematical model accounts for the physical effects of interaction between radio waves and a rough sea surface typical of low grazing angles. This model could be used for generating the input data for the design of radar echo-signal processing algorithms used for solving oceanological problems and ecological monitoring.

REFERENCES

1. Leont'ev V. V., Pimenov A. A. New Paradigm for Solving the Problem of Radar Detection of Oil Films with Sliding Angles of Irradiation of the Sea Surface. Journal of the Russian Universities. Radioelectronics. 2015, vol. 18, no. 6, pp. 46–48. (In Russ.)

2. Lee P. Y., Barter J. D., Beach K. L.; Hindman C. L., Lade B. M., Rungaldier H., Shelton J. C., Williams A. B., Yee R., Yuen H. C. X-band microwave backscattering from ocean waves. J. of geophysical research, 1995, vol. 100, no. 2, pp. 2591–2611. doi: 10.1029/94JC02741

3. Yang P., Guo L., Jia C. Electromagnetic scattering and Doppler spectrum simulation of time-varying oil-covered nonlinear sea surface. J. of Applied Remote Sensing, 2016, vol. 10, no. 1, pp. 1–14. doi: 10.1117/1.JRS.10.016015

4. Wang J., Xu X. Doppler simulation and analysis for 2-D sea surface up to Ku-band. IEEE trans. on Geoscience and Remote Sensing. 2016, vol. GRS-54, no. 1, pp. 466–478. doi: 10.1109/TGRS.2015.2459598

5. Raynal A. M., Doerry A. W. Doppler characteristics of sea clutter. Sandia Report SAND2010-3828, 2010, pp. 27–29. doi: 10.2172/992329

6. Yurovskii Yu. Yu., Malinovskii V. V., Smolov V. E. Radar Methods Of Coastal Zone Monitoring: Opportunities And Problems Of Use. Sevastopol, *Isd-vo Morskogo gidrofisicheskogo instituta NAN Ukrainy*, 2008, 75 p. (Modern problems of oceanology. Vol. 4). (In Russ.)

7. Malinovskii V. V. Estimation of the Relationship of the Parameters of the Radar Signal Reflected from the Sea at Small Slip Angles with the Characteristics of Wind-Wave Collapses. *Morskoi gidrofisicheskii zhurnal* [Marine Hydrophysical Journal], 1991, no. 6, pp. 32–41. (In Russ.)

8. Walker D. Experimentally motivated model for low grazing angle radar Doppler spectra of the sea surface. IEE proc. on Radar, Sonar and Navigation, 2000, vol. RSN-147, no. 3, pp.114–120. doi: 10.1049/ip-rsn:20000386

9. Johnson J. T., Toporkov J., Brown G. A Numerical study of backscattering from time-evolving sea surfaces: comprasion of hydrodynamic models. IEEE trans. on Geoscience and Remote Sensing. 2001, vol. GRS-39, no. 11, pp. 2411–2420. doi: 10.1109/36.964977

10. Toporkov J. K., Brown G. S. Numerical simulations of scattering from time-varying, randomly rough surfaces. IEEE trans. on Geoscience and Remote Sensing. 2000, vol. GRS-38, no. 4, pp. 1616–1624.

11. Rino C. L., Crystal T. L., Koide A. K., Ngo H., Guthart H. Numerical simulation of backscatter from linear and nonlinear ocean surface realisation. Radio Science, 1991, vol. 26, no. 1, pp. 51–71. doi: 10.1029/90RS01687.rf

12. Leont'ev V. V., Pimenov A. A. Justification of the Choice of a Mathematical Model of the Sea Surface When Solving the Problem of Radiolocation Environmental Monitoring. Journal of the Russian Universities. Radioelectronics. 2016, vol. 19, no. 2, pp. 75–79. (In Russ.)

13. Bourlier C., Saillard J., Berginc G. Intrinsic infrared radiation of the sea surface. Progress in electromagnetics research. 2000, vol. 27, pp. 185–335. doi: 10.2528/PIER99080103

14. Shmelev A. B. Wave Scattering by Statistically Uneven Surfaces. *Uspekhi fisicheskikh nauk* [Advances in the Physical Sciences], 1972, vol. 106, no. 3, pp. 459–480. (In Russ.)

15. Bass F. G., Fuks I. M. *Rasseyanie voln na statisticheski nerovnoi poverkhnosti* [Wave Scattering on a Statistically Uneven Surface]. Moscow, *Nauka*, 1972, 424 p. (In Russ.)

16. Skolnik M. *Spravochnik po radiolokatsii* [Handbook of radar]. Vol. 1, Moscow, *Sov. radio*, 1976, 326 p. (In Russ.)

17. Pimenov Yu. V., Vol'man V. I., Muravtsov A. D. *Tekhnicheskaya elektrodinamika* [Technical Electrodynamics]. Moscow, *Radio i svyaz'*, 2000, 536 p. (In Russ.)

18. Tsang L., Kong J. A., Ding K.-H., Ao C. Scattering of Electromagnetic waves: Numerical simulation. New York, John Wiley and Sons, 2001, 716 p. doi:10.1002/0471224308

19. Oh Y., Sarabandi K. Improved numerical simulation of electromagnetic wave scattering from perfectly conducting random surfaces. IEE proc. – microwave antennas propagation. 1997, vol. 144, iss. 4, pp. 256–260. doi: 10.1049/ip-map:19971189

20. Li Y., Wu Z., Zhao J. High-Efficiency numerical computing in low-grazing scattering from sea surface using resistive tapering and forward-backward method. 3rd Asia-Pacific Conference on Antennas and Propagation. 13–16 October 2014, Beijing, China. Bellingham, SPIE, 2014, pp.1101–1104. doi: 10.1109/APCAP.2014.6992702

21. Borodin M. A., Leont'ev V. V., Tret'yakova O. A. Scattering of a Vertically Polarised Electro-Magnetic Wave by a Rough Surface with Gliding Irradiation Journal of the Russian Universities. Radioelectronics. 2010, vol. 13, no. 5, pp. 33–46. (In Russ.)

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СПИСОК ЛИТЕРАТУРЫ

1. Леонтьев В. В., Пименов А. А. Новая парадигма решения задачи радиолокационного обнаружения пленок нефти при скользящих углах облучения поверхности моря // Изв. вузов России. Радиоэлектроника. 2015, № 6. С.46–48.

2. X-band microwave backscattering from ocean waves / P. Y. Lee, J. D. Barter, K. L. Beach; C. L. Hindman, B. M. Lade, H. Rungaldier, J. C. Shelton, A. B. Williams, R. Yee, H. C. Yuen // J. of geophysical research, 1995. Vol. 100, № 2. P. 2591–2611. doi: 10.1029/94JC02741

3. Yang P., Guo L., Jia C. Electromagnetic scattering and Doppler spectrum simulation of time-varying oil-covered nonlinear sea surface // J. of Applied Remote Sensing. 2016. Vol. 10, № 1. P. 1–14. doi: 10.1117/1.JRS.10.016015

4. Wang J., Xu X. Doppler simulation and analysis for 2-D sea surface up to Ku-band // IEEE trans. on Geoscience and Remote Sensing. 2016. Vol. GRS-54, № 1. P. 466–478. doi: 10.1109/TGRS.2015.2459598

5. Raynal A. M., Doerry A. W. Doppler characteristics of sea clutter // Sandia Report SAND2010-3828. 2010. P. 27–29. doi: 10.2172/992329

6. Юровский Ю. Ю., Малиновский В. В., Смолов В. Е. Радиолокационные методы мониторинга прибрежной зоны: возможности и проблемы использования. Севастополь: Изд-во Мор. гидрофиз. ин-та НАН Украины, 2008. 75 с. (Совр. пробл. океанологии. Вып. 4).

7. Малиновский В. В. Оценка связи параметров радиолокационного сигнала, отраженного от моря при малых углах скольжения, с характеристиками обрушений ветровых волн // Мор. гидрофиз. журн. 1991. № 6. С. 32–41.

8. Walker D. Experimentally motivated model for low grazing angle radar Doppler spectra of the sea surface // IEE proc. on Radar, Sonar and Navigation, 2000. Vol. RSN-147, № 3. P.114–120. doi: 10.1049/ip-rsn:20000386

9. Johnson J. T., Toporkov J., Brown G. A Numerical study of backscattering from time-evolving sea surfaces: comprasion of hydrodynamic models // IEEE trans. on Geoscience and Remote Sensing. 2001. Vol. GRS-39, № 11. P. 2411–2420. doi: 10.1109/36.964977

10. Toporkov J. K., Brown G. S. Numerical simulations of scattering from time-varying, randomly rough surfaces // IEEE trans. on Geoscience and Remote Sensing. 2000. Vol. GRS-38, № 4. P. 1616–1624

11. Numerical simulation of backscatter from linear and nonlinear ocean surface realisation / C. L. Rino, T. L. Crystal, A. K. Koide, H. Ngo, H. Guthart // Radio Science, 1991. Vol. 26, № 1. P. 51–71. doi: 10.1029/90RS01687

12. Леонтьев В. В., Пименов А. А. Обоснование выбора математической модели морской поверхности при решении задачи радиолокационного экологического мониторинга // Изв. вузов России. Радиоэлектроника. 2016. № 2. С. 75–79.

13. Bourlier C., Saillard J., Berginc G. Intrinsic infrared radiation of the sea surface // Progress in electromagnetics research. 2000. Vol. 27. P. 185–335. doi: 10.2528/PIER99080103

14. Шмелев А. Б. Рассеяние волн статистически неровными поверхностями // Успехи физ. наук. 1972. Т. 106, вып. 3. С. 459–480.

15. Басс Ф. Г., Фукс И. М. Рассеяние волн на статистически неровной поверхности. М.: Наука, 1972. 424 с.

16. Сколник М. Справочник по радиолокации. Т. 1. М.: Сов. радио, 1976. 326 с.

17. Пименов Ю. В., Вольман В. И., Муравцов А. Д. Техническая электродинамика. М.: Радио и связь, 2000. 536 с.

18. Scattering of Electromagnetic waves: Numerical simulation / L. Tsang, J. A. Kong, K.-H. Ding, C. Ao. New York: John Wiley and Sons, 2001. 716 p. doi:10.1002/0471224308

19. Oh Y., Sarabandi K. Improved numerical simulation of electromagnetic wave scattering from perfectly conducting random surfaces // IEE proc. – microwave antennas propagation. 1997. Vol. 144, iss. 4. P. 256–260. doi: 10.1049/ip-map:19971189

20. Li Y., Wu Z., Zhao J. High-Efficiency numerical computing in low-grazing scattering from sea surface using resistive tapering and forward-backward method // 3rd Asia-Pacific Conference on Antennas and Propagation. 13–16 October 2014, Beijing, China. Bellingham: SPIE, 2014. P.1101–1104. doi: 10.1109/APCAP.2014.6992702

21. Бородин М. А., Леонтьев В. В., Третьякова О. А. Рассеяние вертикально поляризованной электромагнитной волны шероховатой поверхностью при скользящем облучении // Изв. вузов России. Радиоэлектроника. 2010. Вып. 5. С. 33–46.

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