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Original article

Fault Isolation in Network of State Automates

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Abstract

Introduction. In the paper a fault isolation problem in the devices combining digital unit by functional diagnostics methods is considered. Networks of state automates are accepted as mathematical models of the devices. Assumed, that functional diagnostics devices for each network component are preliminarily constructed in an optimal way and they consist of a control automata and of a fault discriminator of unit dimension.

Aim. To develop functional diagnostics method based on theoretical analysis allowing to decide fault isolation problem in networks of state automation and to reduce computational complexity and hardware redundancy.

Materials and methods. An analysis of mathematical description of a network of state automation and functional diagnostics devices for each network component was presented in terms of algebraic theory of functional diagnosis of dynamic systems. A possibility to transform the set of known functional diagnostics devices of the network was demonstrated. The possibility provided a localization of the network component with an error, if the component was unique.

Results. A searching procedure of the analytical equations determining supervision automata and fault discriminator for the whole network was proposed. The case when initial functional diagnostics devices for each network component were defined by scalar functions was considered. The obtained result was generalized to the case, when mentioned devices were defined by vector functions. The application of the described method was demonstrated in the example of construction functional diagnostics devices for simplified fragment of the device for forming priorities of mutual aircraft navigation system.

Conclusion. Estimation of results by an order criterion was obtained. It was established that with an increase in the number of network components, the reduction of intentioned redundancy by functional diagnostics devices compared with the original version increased significantly.

Keywords: diagnostic object, functional diagnosis devices, network of automates, state vector, fault isolation, supervision automata, fault discriminator, compliance function, decision function

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Локализация ошибок в сетях из цифровых автоматов состояний

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Аннотация

Введение. Для повышения достоверности выходной информации систем радиолокации и радионавигации нередко требуется локализация узлов с ошибками в режиме реального времени. Один из самых эффективных способов решения задачи локализации состоит во введении в состав систем средств функционального диагностирования. Однако для систем, имеющих большое количество функциональных узлов, на применение этого способа накладываются ограничения: сложность решения диагностической задачи и необходимость сокращения введенной аппаратной избыточности. Пути редукции этих ограничений при решении задачи локализации в упомянутых системах исследованы в настоящей статье.

Цель работы. Разработка метода синтеза средств функционального диагностирования, решающего задачу локализации ошибок систем радиолокации и радионавигации и позволяющего снизить вычислительную трудоемкость и уменьшить аппаратные затраты.

Материалы и методы. В качестве математических моделей систем приняты сети из цифровых автоматов состояний. Представлен анализ математического описания сети из цифровых автоматов состояний, а также средств функционального диагностирования каждого компонента сети. Показана возможность преобразования совокупности известных средств функционального диагностирования сети, обеспечивающая локализацию компонента сети с ошибкой при условии его единственности.

Результаты. Предложена процедура поиска аналитических выражений, задающих контрольный автомат и дискриминатор ошибок для всей сети. Рассмотрен случай, когда исходные средства функционального диагностирования компонентов заданы скалярными функциями. Полученный результат обобщен на случай векторного задания функций упомянутых средств.

Заключение. Анализ полученных результатов при помощи оценки по критерию порядка показывает, что при увеличении числа компонентов сети выигрыш по избыточности, вносимой средствами функционального диагностирования, по сравнению с исходным вариантом, существенно растет для сети, состоящей из семи компонентов. Возможность практического применения результатов исследования показана на примере решения задачи локализации для упрощенного фрагмента устройства формирования приоритетов системы взаимной навигации летательных аппаратов.

Ключевые слова: объект диагностирования, средства функционального диагностирования, сеть из автоматов, вектор состояний, локализация ошибок, контрольный автомат, дискриминатор ошибок, функция соответствия, решающая функция

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Introduction. The need for fault isolation often arises when operating radar and radio navigation systems, with real-time isolation of such devices coming to the fore as a part of improving output information fidelity. Highly effective method for the problem solution is to include into the systems functional diagnosis (FD) devices allowing to avoid costly shut-down of the system. It makes most sense to use FD in fault-tolerant systems since it significantly reduces redundancy that provides the required reliability.

In technical diagnostics, fault detection is understood as detection of abnormal situation existence, whereas fault isolation also implies its location in the object under diagnostics to the required degree of accuracy. In both cases the list of detectable faults is specified, while in the second one the conditions of their isolation are specified additionally [1]. The detection and isolation tasks are often combined and solved together. In foreign literature the specific concept of Fault Detection and Isolation (FDI) is introduced. In our own literature on FD they are often interpreted as two versions of the general diagnostic task. So, in [2] there is analysis of fault detection and isolation methods based on the use of analytical redundancy of mathematical model of the system. In [3, 4] the problem of FDI for linear discrete systems is solved by means of state observers and special types of Kalman filters. The problems of dynamic system fault isolation are described in [5–7].

The aim of the article is to solve the FD problem for the devices under test which mathematical model is a network of state automates. In modern radar and radio navigation systems the devices for signal processing and generation, control and display most commonly are sets of digital components, connected by trunk lines, i.e. keep within the objects being considered. The latter in its turn are viewed as a special case of networks of discrete time dynamic systems so that diagnostic task solution for the components of radar and radio navigation systems should be considered with regard to dynamic system FD algebraic model [8, 9].

Since a network of finite state automata is also a final state automation, it is possible in theory to solve diagnostic problem for it using the well-known synthesis methods of FD [9, 10]. In practical terms however such a solution encounters two obstacles. Firstly, it is necessary to change from network assignment as a set of assignments of its components and their coupling, with implicit function of the network dy-

namics (transitions), to explicit assigning of the latter which is rather complicated in itself. Secondly, the order (dimension) of the network assigning automata will be greater by several fold than its component orders, and since FD facility synthesis complication increases exponentially at least with the order increase, the problem solution becomes impossible due to excessive computational efforts.

For the specified reasons, the FD devices have to be synthesized for each component separately and the FD facilities have to be considered a composition of all the FD facilities components. In this case, fault detection is guaranteed, randomly distributed in the net-work including the ones distorting operating results of all the components equally. Moreover, the faulted components are easily localized. However such a solution often appears redundant since fault probability in several components at a time is quite low. Thus the principal task of this article is to develop the functional diagnosis method for state automation networks which allows to decrease the introduced redundancy by means of rejection of low-probability faults with regard to the specified cases. Further on, there provided simple and effective algorithms for the solution of digital networks FD problem providing minimality of solution in terms of the order.

Diagnostic object. As it has already been mentioned, in this article it is digital automation networks that is considered a device under test. For simplicity, the network components are the automates without output logic converter, i.e. state automates. In this case, each i -th automata-component of the network is given by the triple of $A_i = (X_i, Q_i, \delta_i)$, where $X_i = \{x_i\}$ – a set of automata inputs; $Q_i = \{q_i\}$ – a set of its states; $\delta_i = \delta_i(x_i, q_i)$ – a next-stage function of A_i , with X_i and Q_i being a set of binary vector; δ_i – a vector Boolean (logic) function [11].

The net under diagnostic made as A_i composition (Fig. 1), is also a digital state automation $S = (X, Q, \delta)$. A set of the network states is formed by Cartesian product of all sets of its component states, i.e. $Q = \times_{i=1}^n Q_i$, where n – a number of S components, and vector $q \in Q$ is a vector of the type $q = (q_1, \dots, q_i, \dots, q_n)$. Network input vector

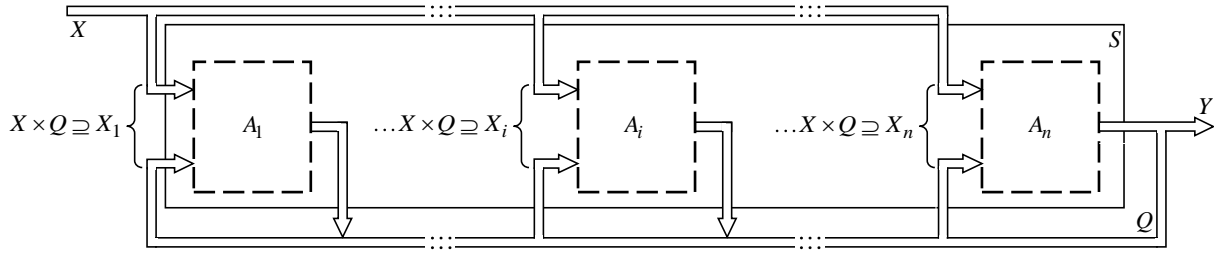


Рис. 1. Диагностируемая сеть

Fig. 1. Network under diagnostic

$\mathbf{x} \in X$ is formed similar to its state vector but includes only external (independent of \mathbf{q}) components \mathbf{x}_i , hence the power of X set is smaller than the power of cartesian product $\prod_{i=1}^n X_i$. Vector boolean next-state function of the network is a composition of all next-state functions of the components: $\delta = (\delta_1, \dots, \delta_i, \dots, \delta_n)$ [12].

Setting a task of diagnostics. Suppose each automata A_i of the network S has diagnostic tools built in the optimal way according to FD base form [13]. Their structure contains:

- device under test $A_i = (X_i, Q_i, \delta_i)$;
- supervision automata $A_{Ki} = (X_{Ki}, Q_{Ki}, \delta_{Ki})$,

which operates synchronously with automata A_i ;

- inertialess functional converter called fault discriminator D_i . The latter consists of two devices: calculator of vector compliance function $\mathbf{r}_i(\mathbf{q}_i)$ and a comparison device \otimes , at the output of which binary decision function is formed $\varepsilon_i[\mathbf{r}_i(\mathbf{q}_i), \mathbf{q}_{Ki}]$ (Fig. 2).

Supervision automata $A_{Ki} = (X_{Ki}, Q_{Ki}, \delta_{Ki})$ monitors the state of the original automata $A_i = (X_i, Q_i, \delta_i)$ with an accuracy sufficient to detect in the last of these the faults of the specified class E_i , with A_{Ki} input being a composition of elements $\mathbf{x}_i \in X_i$ and $\mathbf{q}_i \in Q_i$.

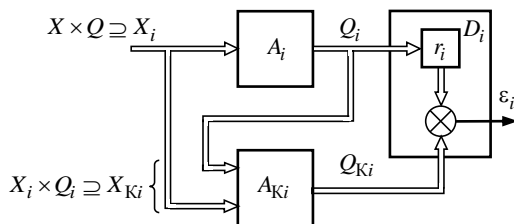


Рис. 2. Устройство сравнения

Fig. 2. Comparison device

The vector length $\mathbf{q}_{Ki} \in Q_{Ki}$ does not exceed the length of $\mathbf{q}_i \in Q_i$, and a set of supervision automata inputs A_{Ki} is a subset of cartesian product $X_i \times Q_i$, where $X_i \subseteq X \times Q$, then $X_{Ki} \subseteq X \times Q \times Q_i$, i.e. generally the next-state function δ_{Ki} in addition to \mathbf{q}_{Ki} , depends on three more arguments external to A_{Ki} : \mathbf{x} , \mathbf{q} и \mathbf{q}_i .

Fault discriminator D_i is a logic scheme where compliance function is calculated $\mathbf{r}_i(\mathbf{q}_i)$, the value of which always equals to $\mathbf{q}_{Ki} \in Q_{Ki}$ in the absence of faults in A_i и A_{Ki} .

When a fault of the specified class occurs the mentioned equality is violated which allows it to be detected. For this purpose a binary decision function is generated in comparison device $\varepsilon_i[\mathbf{r}_i(\mathbf{q}_i), \mathbf{q}_{Ki}]$, the value of which is inverted when the ratio is not valid $\mathbf{r}_i(\mathbf{q}_i) = \mathbf{q}_{Ki}$. It is generally assumed that it is to equal to zero in the absence of faults and to one when the fault occurs.

Since diagnosis devices are built for each network component specifically the fault detection is followed by their localization with an accuracy to the network component, and the faults in all the components can be localized at the same time.

The network considered S , as well as its components A_i , represents digital state automation so the solution of FD task for it can be found in the basic formula. The latter, similar to the previous one (Fig.2) is to include the diagnostic object S , supervision system (automata) S_K and fault discriminator D . The network S consisting of all A_i , is defined, S_K and D need to be synthesized transforming the sets of the known A_{Ki} and D_i in S_K and D respectively, where the method of transformation signifi-

cantly depends on the class of faults to be detected and localized.

Note that the mentioned class cannot be wider than aggregation of fault classes localized by means of A_{Ki} and D_i , and the reduction in costs for FD can occur only due to its narrowing.

On the basis of the foregoing, the problem considered in this article can be posed as follows.

Let n digital state automates A_i form a network S and for each automata of the network the FD task diagnostic task is optimally solved in the basic form, i.e. sets of supervision automates A_{Ki} and fault discriminators D_i are synthesized. In the same form it is necessary to build the FD devices for the entire network so that the included control system S_K has a minimum order and together with the fault discriminator D ensures fault detection and isolation in an arbitrary but the single network component.

FD problem solution. To solve the problem it is necessary to find three logical functions determining the type of the supervision system S_K and fault discriminator D . First, it is a compliance function $\mathbf{r}(\mathbf{q})$. In general it is a vector function and on the one part it is synthesized based on the class of localized faults, and on the other part it specifies the state vector S_K , as in the absence of faults in the network $\mathbf{q}_K = \mathbf{r}(\mathbf{q})$. Secondly, the decision function $\boldsymbol{\varepsilon}[\mathbf{r}(\mathbf{q}), \mathbf{q}_K]$, which characterizes the method of fault detection and depth of its localization. When the fault is localized it can be scalar even in case of multiple faults [14], localization necessarily requires its vector. Thirdly, it is the $\delta_K(\mathbf{x}_K, \mathbf{q}_K)$ next-state function of the system S_K , also generally vector one. The first two functions specify the fault discriminator D_i , the second and the third ones – the supervision system S_K .

First of all, transforming the known set of decision functions for A_i , let's find the decision function $\boldsymbol{\varepsilon}$. It depends on the state vector \mathbf{q}_K and compliance function $\mathbf{r}(\mathbf{q})$, with the latter to explicitly distinguish from the expression for $\boldsymbol{\varepsilon}$. For this purpose it is necessary to represent $\boldsymbol{\varepsilon}$ in the form of $\mathbf{r}(\mathbf{q})$ and \mathbf{q}_K divisible decomposition that while solving the network FD problems can be obtained only using linear transformations [14, 15].

We will find the required transformation using the uniqueness condition of the network component with an error. It follows that the initial vector decision function $\boldsymbol{\varepsilon}_n = (\boldsymbol{\varepsilon}_1, \dots, \boldsymbol{\varepsilon}_i, \dots, \boldsymbol{\varepsilon}_n)$, the components of which being decision functions of all A_{Ki} , equals to zero vector in the absence of errors and to the Hamming unit norm [16] if they exist, with the number of the latter being n .

With zeroes and ones we form the G matrix with $m \times n$ dimensions, where $m = \lceil \log_2(n+1) \rceil$ is the nearest to $\log_2(n+1)$ larger integer, so that its rows represent consecutive m -bit binary vectors (numbers) from 1 to m , and find the product of $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_n G$. The m -bit vector $\boldsymbol{\varepsilon}$ and n -bit vector $\boldsymbol{\varepsilon}_n$ values are in one-to-one relation, and with the unit norm of $\boldsymbol{\varepsilon}_n$ the value of the binary vector $\boldsymbol{\varepsilon}$ coincides with the number of on-bit $\boldsymbol{\varepsilon}_n$. If there are no errors in the S network $\boldsymbol{\varepsilon}_n = 0$ and vector $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_n G$ equals to zero as well which is representative of the system normal operation.

The transformation introduced allows us to localize an automaton with an error in the network at the minimum dimension of the vector fixing them and determines the desired vector decision function as the result of multiplying the vector argument formed by the composition of scalar functions ε_i by the matrix G introduced above:

$$\boldsymbol{\varepsilon} = (\varepsilon_1, \dots, \varepsilon_i, \dots, \varepsilon_n) G, \quad (1)$$

Relation (1), specifying the decision function through a linear transformation, however, is not a separable decomposition. Let us consider the possibility of changing its shape. To begin with, we assume that the arguments of all the decision functions ε_i are scalar, i.e.

$$\varepsilon_i[\mathbf{r}_i(\mathbf{q}_i), \mathbf{q}_{Ki}] = \varepsilon_i[r_i(\mathbf{q}_i), q_{Ki}].$$

For example, this situation is observed if only single fold errors are found in all A_i . Since in the scalar case $\varepsilon_i[r_i(\mathbf{q}_i), q_{Ki}] = r_i(\mathbf{q}_i) \oplus q_{Ki}$ [14], a simple substitution transforms (1) into

$$\begin{aligned} \boldsymbol{\varepsilon} &= (\varepsilon_1, \dots, \varepsilon_i, \dots, \varepsilon_n) G = \\ &= [r_1(\mathbf{q}_1) \oplus q_{K1}, \dots, r_i(\mathbf{q}_i) \oplus q_{Ki}, \dots, \\ &\quad \dots, r_n(\mathbf{q}_n) \oplus q_{Kn}] G, \end{aligned}$$

which, after taking into account the associativity of modulo 2 addition and the linearity of matrix multiplication, will result in definition of the decision function as the component-wise sum of two vectors of the form

$$\mathbf{\varepsilon} = [\eta(\mathbf{q}_1), \dots, r_i(\mathbf{q}_i), \dots, r_n(\mathbf{q}_n)]G \oplus (q_{K1}, \dots, q_{Ki}, \dots, q_{Kn})G. \quad (2)$$

The resulting amount is a separable decomposition from which the state vector \mathbf{q}_K and the compliance function $\mathbf{r}(\mathbf{q})$ are easily distinguished:

$$\begin{cases} \mathbf{q}_K = (q_{K1}, \dots, q_{Ki}, \dots, q_{Kn})G; \\ \mathbf{r}(\mathbf{q}) = [\eta(\mathbf{q}_1), \dots, r_i(\mathbf{q}_i), \dots, r_n(\mathbf{q}_n)]G. \end{cases} \quad (3)$$

In the absence of errors in S and S_K we have $\mathbf{q}_K = \mathbf{r}(\mathbf{q})$. The appearance of errors violates this equality, which allows them to be detected and localized.

To complete the setting of the control system S_K , using the whole of transition functions of the automata A_{Ki} , we need to find the transition function $\delta_K(\mathbf{x}_K, \mathbf{q}_K)$. For compactness, let us denote the vector which components are the transition functions of these automata by the symbol $\langle \delta_{Ki}(\mathbf{x}_{Ki}, \mathbf{q}_{Ki}) \rangle$. If all the compliance functions $r_i(\mathbf{q}_i)$ are scalar, all components of this vector are also scalar: $\delta_{Ki}(\mathbf{x}_{Ki}, q_{Ki}) = \delta_{Ki}(\mathbf{x}_{Ki}, q_{Ki})$. Since the states and the transition function of the automaton A_{Ki} in two adjacent clock cycles t and t^* are connected by the equality $q_{Kit^*} = \delta_{Ki}(\mathbf{x}_{Kit}, q_{Kit})$, it follows from (3) that in a clock cycle t^* the vector $\mathbf{q}_{Kt^*} = \langle \delta_{Ki}(\mathbf{x}_{Kit}, q_{Kit}) \rangle G$ and further

$$\begin{aligned} \delta_K(\mathbf{x}_K, \mathbf{q}_K) &= \langle \delta_{Ki}(\mathbf{x}_{Ki}, q_{Ki}) \rangle G = \\ &= [\delta_{K1}(\mathbf{x}_{K1}, q_{K1}), \dots, \delta_{Ki}(\mathbf{x}_{Ki}, q_{Ki}), \dots, \\ &\quad \dots, \delta_{Kn}(\mathbf{x}_{Kn}, q_{Kn})]G. \end{aligned} \quad (4)$$

The resulting expression specifies the transition function S_K , however, it is not constructive since in its right-hand side it contains A_{Ki} states which you need to get rid of in the final solution. To do this, using the equality $r_i(\mathbf{q}_i) = q_{Ki}$, which is valid in the absence of errors, we replace the states q_{Ki} in (4)

with the compliance functions $r_i(\mathbf{q}_i)$ and obtain $\delta_K(\mathbf{x}_K, \mathbf{q}_K) = \langle \delta_{Ki}[\mathbf{x}_{Ki}, r_i(q_i)] \rangle G$. The right-hand side of this equality does not depend on \mathbf{q}_K , that determines the implementation of the S_K system in the form of a logical delay [8] and allows us to finally find its transition function as

$$\begin{aligned} \delta_K(x_K) &= \langle \delta_{Ki}[\mathbf{x}_{Ki}, r_i(q_i)] \rangle G = \\ &= \{ \delta_{Ki}[\mathbf{x}_{Ki}, r_i(q_i)], \dots, \delta_{K1}[\mathbf{x}_{K1}, r_1(q_1)], \dots, \\ &\quad \dots, \delta_{Kn}[\mathbf{x}_{Kn}, r_n(q_n)] \} G. \end{aligned} \quad (5)$$

The vector \mathbf{x}_K in (5) is composite. In the general case, in addition to the components of the input vector of the system S , it includes the components of both its state vector and the state vector of all A_i . This is explained by the fact that from $X_{Ki} \subseteq X \times Q \times Q_i$ it fol-

lows that $X_K \subseteq X \times Q \times \left(\times_{i=1}^n Q_i \right)$, and $\mathbf{x}_K \in X_K$.

Thus, since all 3 functions defining the control system S_K , and the fault discriminator D are determined by relations (2), (3) and (5), and the method for generating the matrix G included in them is specified, the problem posed as applied to the case of scalarity of all compliance functions for the network S components is solved. The control system S_K is implemented in the form of logical delay in a manner that is minimal according to the criterion of order. Technically, its construction is reduced to solving the problem of logical synthesis by means of standard methods [9, 10].

If the compliance functions $r_i(\mathbf{q}_i)$ are vectorial for solving the problem of fault isolation in the network S , the previously proposed synthesis method S_K should be changed in a similar manner to the modification of the procedure for constructing FD devices for fault detection used in [14]. We will find the necessary changes, assuming that the dimension of all the compliance functions is the same and equals to p . In this case, each of them is a vector the components of which are scalar functions:

$$r_i(\mathbf{q}_i) = [r_{i1}(\mathbf{q}_i), \dots, r_{ij}(\mathbf{q}_i), \dots, r_{ip}(\mathbf{q}_i)],$$

and the state vector A_{Ki} takes the form of

$$\mathbf{q}_{Ki} = [q_{Ki1}, \dots, q_{Kij}, \dots, q_{Kip}].$$

In the absence of errors, these vectors are termwise equal:

$$\mathbf{q}_{Ki} = \mathbf{r}_i(\mathbf{q}_i) = [q_{Ki1} = r_{i1}(\mathbf{q}_i), \dots, q_{Kij} = r_{ij}(\mathbf{q}_i), \dots, q_{Kip} = r_{ip}(\mathbf{q}_i)].$$

Then the decision function takes the form of a logical

sum ("OR" operation) $\varepsilon_i = \bigvee_{j=1}^p \varepsilon_{ij}$, where

$$\varepsilon_{ij} = r_{ij}(\mathbf{q}_i) \oplus q_{Kij}.$$

As before, the assumed vector decision function of dimension n for the network S is specified by the expression $\varepsilon_n = (\varepsilon_1, \dots, \varepsilon_i, \dots, \varepsilon_n)$, substituting into which the logical sums defining ε_i , we obtain

$$\varepsilon_n = \left(\bigvee_{j=1}^p \varepsilon_{1j}, \dots, \bigvee_{j=1}^p \varepsilon_{ij}, \dots, \bigvee_{j=1}^p \varepsilon_{nj} \right). \quad (6)$$

Having formed auxiliary vector functions of the form $\varepsilon_{nj} = (\varepsilon_{1j}, \dots, \varepsilon_{ij}, \dots, \varepsilon_{nj})$, we can verify that the decision function given by the relation (6) is the componentwise logical sum of all ε_{nj} . Obviously, each

ε_{nj} provides the detection and isolation of some of the faults in S (E_j subclass)

$(q_{K1j}, \dots, q_{Kij}, \dots, q_{Knj})G$, and their combination

– all faults of the specified class $E \bigcup_{j=1}^p E_j$.

Let us choose a certain function ε_{nj} and solve the localization problem as applied to the faults from the subclass E_j , assuming that they occur in one and only one component S . Since all its components $\varepsilon_{ij} = r_{ij}(\mathbf{q}_i) \oplus q_{Kij}$ are formed as sums of scalars, the desired solution does not differ from the one obtained before, and the FD devices in the form of S_{Kj} control subsystem with the fault discriminator D_j are defined by the relations (2), (3) and (5):

$$\varepsilon_j = [r_{1j}(\mathbf{q}_1), \dots, r_{ij}(\mathbf{q}_i), \dots, r_{nj}(\mathbf{q}_n)]G \oplus \oplus (q_{K1j}, \dots, q_{Kij}, \dots, q_{Knj})G;$$

$$\mathbf{q}_{Kj} = (q_{K1j}, \dots, q_{Kij}, \dots, q_{Knj})G;$$

$$r_j(\mathbf{q}) = [r_{1j}(\mathbf{q}_1), \dots, r_{ij}(\mathbf{q}_i), \dots, r_{nj}(\mathbf{q}_n)]G;$$

$$\begin{aligned} \delta_{Kj}(x_K) &= \langle \delta_{Kij} [x_{Ki}, r_{ij}(q_i)] \rangle G = \\ &= \{ \delta_{K1j} [x_{K1}, r_{1j}(q_1)], \dots, \delta_{Kij} [x_{Ki}, r_{ij}(q_i)], \dots, \\ &\dots, \delta_{Knj} [x_{Kn}, r_{nj}(q_n)] \} G. \end{aligned}$$

When forming the decision function for the entire network S , it should be taken into account that the fault subclasses forming E can intersect, since if the compliance functions are vector, the error in a certain network component A_i can violate the equality $\mathbf{q}_{Ki} = \mathbf{r}_i(\mathbf{q}_i)$ in several components at a time, i.e., under the sign of the i -th logical sum in (6) there can be several unit summands. It will bring into existence of several functions among ε_{nj} the values of which are equal with the given fault and the indices are different. Then there will be several such functions among ε_j , which excludes the possibility of using their component-wise summation modulo 2 when forming the desired decision function, since the result of such summation can be a zero vector corresponding to the absence of errors.

In this case, the logical summation ("OR" operation) should be used. Then the desired decision function takes the following form:

$$\begin{aligned} \varepsilon &= \bigvee_{j=1}^p \varepsilon_j = \\ &= \bigvee_{j=1}^p \{ [r_{1j}(\mathbf{q}_1), \dots, r_{ij}(\mathbf{q}_i), \dots, r_{nj}(\mathbf{q}_n)]G \oplus \\ &\oplus (q_{K1j}, \dots, q_{Kij}, \dots, q_{Knj})G \}. \end{aligned} \quad (7)$$

Since (7) is not separable by components $\mathbf{r}(\mathbf{q})$ and \mathbf{q}_K , and the compliance function $\mathbf{r}(\mathbf{q})$ is formed as a composition of all functions $\mathbf{r}_j(\mathbf{q})$ and consists of pm scalar components:

$$\begin{aligned} \mathbf{r}(\mathbf{q}) &= \{ [r_{11}(\mathbf{q}_1), \dots, r_{n1}(\mathbf{q}_n)]G, \dots, \\ &\dots, [r_{1j}(\mathbf{q}_1), \dots, r_{nj}(\mathbf{q}_n)]G, \dots, \\ &\dots, [r_{1p}(\mathbf{q}_1), \dots, r_{np}(\mathbf{q}_n)]G \}. \end{aligned} \quad (8)$$

Based on the equality $\mathbf{q}_K = \mathbf{r}(\mathbf{q}_i)$, we obtain the expressions for \mathbf{q}_K and $\delta_K(x_K)$:

$$\begin{aligned} \mathbf{q}_K &= [(q_{K11}, \dots, q_{K1l}, \dots, q_{K1n})G, \dots, \\ &\dots, (q_{K1j}, \dots, q_{Kij}, \dots, q_{Knj})G, \dots, \\ &\dots, (q_{K1p}, \dots, q_{Kip}, \dots, q_{Knp})G]; \end{aligned} \quad (9)$$

$$\begin{aligned} \delta_K(\mathbf{x}_K) = & \left\{ \delta_{K11}[\mathbf{x}_K, r_{11}(\mathbf{q}_1)], \dots \right. \\ & \dots, \delta_{Knl}[\mathbf{x}_{Kn}, r_{nl}(\mathbf{q}_n)] \left. \right\} G, \dots \\ & \dots, \left\{ \delta_{K1j}[\mathbf{x}_{K1}, r_{1j}(\mathbf{q}_1)], \dots \right. \\ & \dots, \delta_{Knj}[\mathbf{x}_{Kn}, r_{nj}(\mathbf{q}_n)] \left. \right\} G, \dots \\ & \dots, \left\{ \delta_{K1p}[\mathbf{x}_{K1}, r_{1p}(\mathbf{q}_1)], \dots \right. \\ & \dots, \delta_{Knp}[\mathbf{x}_{Kn}, r_{np}(\mathbf{q}_n)] \left. \right\} G. \end{aligned} \quad (10)$$

The dimensions of the state vector \mathbf{q}_K and the transition function $\delta_K(\mathbf{x}_K)$ coincide with the dimensions of the compliance function $\mathbf{r}(\mathbf{q})$ and equal to $pm = p \lceil \log_2(n+1) \rceil$.

The relations (7)–(10) completely specify the control system S_K and the fault discriminator D for the network S in the case of equal dimension of the vector compliance functions of its components. It is easy to verify that with the unit dimension of these functions (the functions are scalar) (7) and (10) take the form of relations (2) and (5) respectively, and (8) and (9) are transformed into the right-hand side and left-hand side relations (3).

If the dimensions of the compliance functions $\mathbf{r}_i(\mathbf{q}_i)$ are not equal, then for the synthesis of FD devices, the obtained relations can be used provided that the functions of lower dimension are supplemented with zero components to the maximum value, while the corresponding state vectors \mathbf{q}_{Kj} are also supplemented with zeros.

Estimated results. We will estimate the results obtained by comparing the complexity of the initial FD devices with the ones obtained as a result of the proposed transformations. To estimate, we use the order criterion [17], the value of which in the case under consideration equals the total dimension of the state vectors of all A_{Ki} for the initial version and the dimension \mathbf{q}_K for the transformed one. If the compliance functions $\mathbf{r}_i(\mathbf{q}_i)$ are scalar, the first is n , and the second is $\lceil \log_2(n+1) \rceil$, i.e., the gain in order

$$\eta = n / \lceil \log_2(n+1) \rceil. \quad (11)$$

The estimate (11) is also valid when the compliance functions are vector but have the same dimension p , since in this case the dimensions of all state vectors increase p times, i.e., the numerator and de-

nominator (10) grow proportionally and the value of η does not change. In the case of different dimensions of the compliance functions $\mathbf{r}_i(\mathbf{q}_i)$, the calculation of η value is not complex either, but there is no such compact formula as of the form (11).

Analyzing (11), we can conclude that the method for synthesizing FD devices presented in this article for a network of state automates with compliance functions of the similar dimension gives a gain if the number of its components $n \geq 3$. With an increase in the number of network components, the gain grows quickly but not monotonously: η has local maxima at $n = 2^k - 1$ and minima at $n = 2^k$ ($k \geq 2$ is an integer). In the case of different dimensions of the compliance functions, this growth continues but its intensity depends on the distribution of dimensions over them. In the worst case, when only one function has p dimension and the others have 1, the gain is observed only when $p \geq \lceil \log_2(n+1) \rceil$.

Example. Let us consider a simplified fragment of a priority driver of the aircraft mutual navigation system. A fragment is a network with individual FD component devices the operational part of which consists of two binary counters and one shift register, and the diagnostic part consists of five D type triggers with calculators of compliance functions and decision functions (Fig. 3). The fragment simplification is reducing the number of bits of the operating part to the level that allows to obtain its compact representation.

FD devices provide detection and isolation of single errors: arithmetic errors in counters and group ones in the register. Using the described technique, we convert them into network devices of the minimum order FD that guarantee the detection and isolation of the same errors, provided that they exist only in one component of the network.

1. In the case under consideration $n=3$ and $m = \lceil \log_2(n+1) \rceil = 2$, and the transforming matrix

$$G = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

2. Functions $\mathbf{r}_1(\mathbf{q}_1) = (r_{11}, r_{12})$ и $\mathbf{r}_2(\mathbf{q}_2) = (r_{21}, r_{22})$ are vector, and $r_3(\mathbf{q}_3) = r_{31}$ is scalar (Fig. 3), therefore, the desired network com-

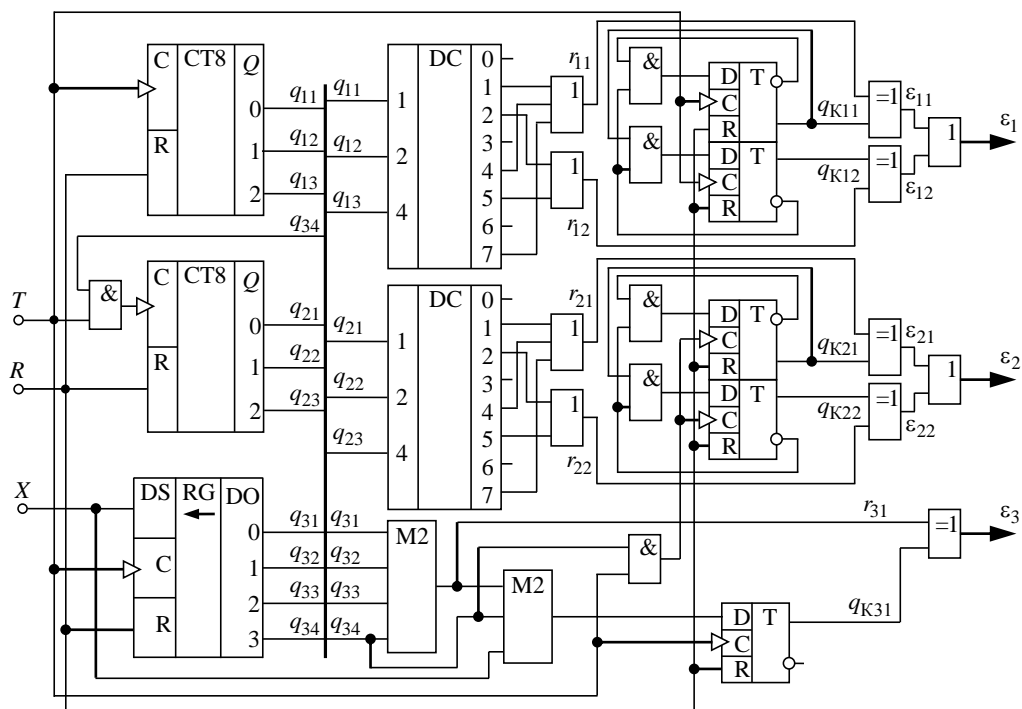


Рис. 3. Упрощенная принципиальная схема фрагмента формирователя приоритетов системы взаимной навигации летательных аппаратов

Fig. 3. A simplified schematic diagram of the fragment of priority driver of the aircraft mutual navigation system

pliance function is calculated by formula (8) after adding to r_{31} zero component:

$$\mathbf{r}(\mathbf{q}) = [(\eta_1, r_{21}, r_{31})G, (\eta_2, r_{22}, 0)G] = \\ = (r_1 = r_{21} \oplus r_{31}, r_2 = \eta_1 \oplus r_{31}, r_3 = r_{22}, r_4 = \eta_2).$$

3. According to (9)

$$\mathbf{q}_K = (q_{K1} = q_{K21} \oplus q_{K31}, q_{K2} = q_{K11} \oplus q_{K31}, \\ q_{K3} = q_{K22}, q_{K4} = q_{K12}),$$

i.e., the state vector \mathbf{q}_K is a four-bit binary vector.

4. For implementation A_K in the form of logical delay, using (10) and omitting the arguments of the transition functions, we obtain:

$$\delta_K = (\delta_{K1} = \delta_{K21} \oplus \delta_{K31}, \delta_{K2} = \delta_{K11} \oplus \delta_{K31}, \\ \delta_{K3} = \delta_{K22}, \delta_{K4} = \delta_{K12}).$$

Having determined δ_{Kij} through the analysis of Fig. 3, taking into account the equality $q_{Kij} = r_{ij}$ we finally obtain:

$$\delta_K = [\delta_{K1} = \bar{r}_{22}(q_{34} \oplus r_{21}) \oplus X \oplus r_{31} \oplus q_{34}, \\ \delta_{K2} = \bar{r}_1 \bar{r}_{12} \oplus X \oplus r_{31} \oplus q_{34}, \\ \delta_{K3} = \bar{q}_{34} r_{22} \bar{r}_{21} \vee q_{34} \bar{r}_{22} r_{21}, \delta_{K4} = r_1 \bar{r}_{12}].$$

5. The vector decision function is determined in accordance with (7):

$$\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2) = \\ = [(r_{21} \oplus r_{31} \oplus q_{K21} \oplus q_{K31}) \vee (r_{22} \oplus q_{K22}), \\ (\eta_1 \oplus r_{31} \oplus q_{K11} \oplus q_{K31}) \vee (\eta_2 \oplus q_{K12})] = \\ = [(r_1 \oplus q_{K1}) \vee (r_3 \oplus q_{K3}), \\ (r_2 \oplus q_{K2}) \vee (r_4 \oplus q_{K4})] = \\ = (\varepsilon_{11} \vee \varepsilon_{12}, \varepsilon_{21} \vee \varepsilon_{22}).$$

Thus, all the functions that determine S_K and D are obtained, which allows using the standard methods for the synthesis of digital devices to build the desired FD devices (Fig. 4).

Comparison of the FD devices synthesized by means of the proposed method (Fig. 4) with the initial ones (Fig. 3) shows that for almost the same logical costs, the first-order (memory) gain is 25%. This relatively small value confirms the validity of the above estimations of the method effectiveness and is explained by the small dimension of the network S ($n=3$) and different dimension of the compliance functions of the network components.

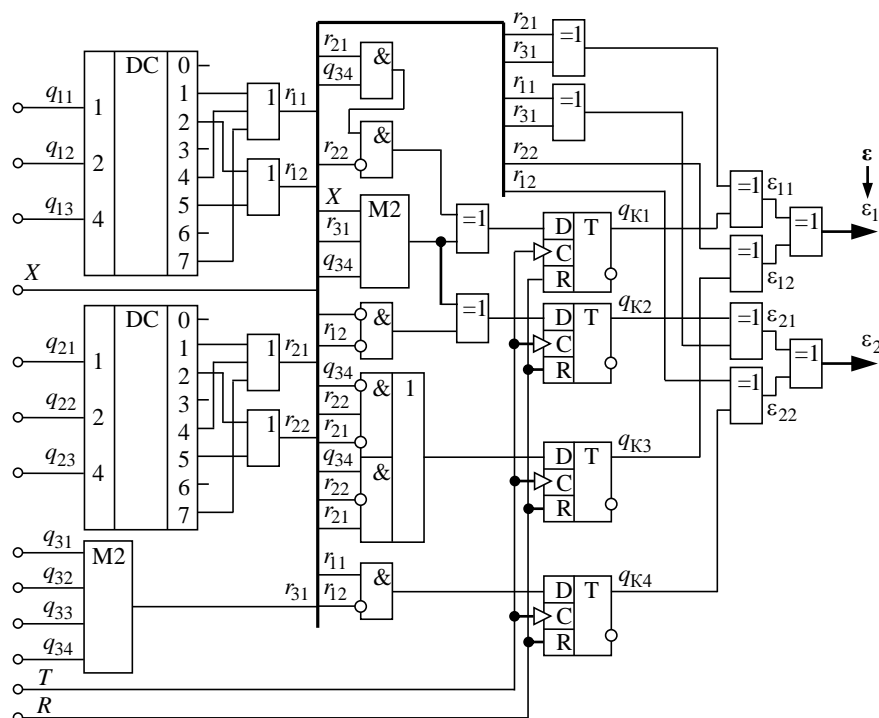


Рис. 4. Синтезированные средства функционального диагностирования
Fig. 4. Synthesized equipment of the functional diagnostics

Conclusion. In this article, the diagnostic problem for networks of state automates is solved to an accuracy of isolation of the fault component. The proposed method for its solution reduces computational complexity as well as the amount of diagnostic equipment in comparison with the methods known from the literature

sources. The estimates obtained allow us to determine the efficiency and feasibility of the transformations introduced before solving the problem of synthesis of FD devices. In the future, the results described in this article are meant to be generalized for the case of network of state automates of a general form.

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