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ОБЗОРНАЯ СТАТЬЯ

Радиотехнические средства передачи, приема и обработки сигналов
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REVIEW ARTICLE

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Retrospective Review of Perfect Ternary Sequences and Their Generators

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Abstract

Introduction. Perfect polyphase unimodular sequences, i.e. sequences with ideal periodic autocorrelation and single ampli-tude of symbols are widely used in modern radio communications and radars. Among them a special place is occupied by perfect ternary sequences (PTSs) with elements {–1, 0, 1}. PTSs are quite numerous and their length in comparison with perfect binary sequences is not limited from above. There is a well-known review of PTS families undertaken by Fan and Darnell in 1996. However, over the past two decades numerous new PTS families have been discovered. Connections between PTSs and circulant weighing matrices have been es-tablished and certain theorems on the existence of PTS existence for certain lengths have also been obtained. Therefore, there is a need for a new modern review of existing PTSs.

Objective. This review of existing PTSs is intended for developers of radio electronic systems using perfect sequences.

Materials and methods. Domestic and foreign sources of information (books, journal papers, conference proceedings, patents) were considered and analysed. A Web search was carried out based on keywords using resources of Yandex and Google, as well as in digital electronic libraries (Russian State Library (RSL), IEEE Xplore Digital Library), conference materials (Digital Signal Processing and its Application (DSPA), Sequences and their Applications (SETA), etc.).

Results. In addition to the matter of collating an informational bibliography, the review shows the relationship be-tween PTSs obtained at different times and their connection with circulant weighing matrices. The review also describes the generators of known PTS families (Ipatov, Hoholdt-Justensen, etc.).

Conclusion. A retrospective review of PTSs is herein presented and the generators of certain known PTS families have been considered. The results of the study are relevant for use in modern radio communications and radar sys-tems and in CW and LPI radars in particular.

Key words: radio signals, perfect ternary sequences, sequence generators

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REVIEW ARTICLE

Радиотехнические средства передачи, приема и обработки сигналов
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Ретроспективный обзор троичных последовательностей с идеальной периодической автокорреляцией и устройств их генерации

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Аннотация

Введение. Идеальные многофазные унимодулярные последовательности, т. е. последовательности с идеальной периодической автокорреляцией и единичной амплитудой символов, широко используются в современной радиосвязи и радиолокации. Особое место среди них занимают идеальные троичные последовательности (ИТП) с элементами {–1, 0, 1}. ИТП достаточно многочисленны, а их длина в отличие от идеальных двоичных последовательностей не ограничена сверху. Известен обзор ИТП, сделанный Фаном и Дарнеллом в 1996 г. Однако за прошедшие два десятилетия были открыты новые многочисленные семейства ИТП, установлены связи между ИТП и циркулянтными взвешенными матрицами, получены теоремы о существовании ИТП с определенными параметрами. Поэтому возникла потребность в новом современном обзоре известных на сегодня ИТП.

Цель работы. Обзор современных ИТП предназначен для разработчиков радиоэлектронных систем, в которых используются идеальные последовательности.

Материалы и методы. Рассмотрены и проанализированы отечественные и зарубежные источники информации (книги, журнальные статьи, труды конференций, патенты). Поиск осуществлялся в сети Интернете по ключевым словам с использованием Интернет-ресурсов Yandex и Google, а также в цифровых электронных библиотеках (Российской Государственной библиотеке (РГБ), IEEE Xplore Digital Library), в материалах конференций (Цифровая Обработка Сигналов и ее Применение (DSPA), Sequences and Their Applica-tions (SETA), и др.).

Результаты. Наряду с решением информационно-библиографической задачи в обзоре показана взаимосвязь полученных в разное время ИТП, их эквивалентность циркулянтным взвешенным матрицам, а также рассмотрены устройства генерации известных семейств ИТП (Ипатова, Хохолдта-Джастесена и др.).

Заключение. Представлен ретроспективный обзор ИТП; рассмотрены генераторы известных семейств ИТП. Результаты исследования актуальны для применения в современных системах радиосвязи и радиолокации, в частности в CW- и LPI-радарах.

Ключевые слова: радиосигналы, идеальные троичные последовательности, генераторы последовательностей

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Introduction. Perfect polyphase unimodular sequences, i.e. sequences with an ideal periodic autocorrelation and unit amplitude of symbols are widely used in modern radio communications and radars [1, 2, 3, 4]. Their application in continuous wave radars (CW-radars) [3] and in low probability intercept radars [4] is the most promising. The most well-known polyphase sequences are the Frank, Zadoff-Chu and Milewski perfect polyphase sequences and their numerous modifications [2, 3]. However, the general property of all these sequences is an increase in alphabet volume with an increase of their length.

At the same time, there are various families of perfect polyphase sequences with zeroes whose alphabet volume does not depend on their length. The charge for this is a peak factor greater than one, causing energy losses in the receiver. The 4-phase general Lee sequences [5] with length $\left(p^m+1\right)$, where $p>2$ is a prime; $m\geq 1$ is an integer, and $\left(p^m+1\right)\equiv 2\bmod 4$; the 8-phase Lüke sequences with length $\left(p^n-1\right)/\left(p^m-1\right)\equiv 4\bmod 8$, where $p>2$ is a prime, $n\geq 2$ is an integer, $m\geq 1$ is an integer , and $m|n$ [6] (term $m|n$ means, that m is a factor of n); 4-phase Schotten-Lüke sequence with length $\left(p^n-1\right)/\left(p^m-1\right)$, where $p=4t+1$ is a prime, $n>1$ and m are integers, $m|n$ and $n\neq 2m$ [7]; and also 4- and 8-phase sequences [8] with length $N=2\left(p^n-1\right)/\left(p^m-1\right)$, where $p>2$ is a prime, $n=mk$, $m\geq 1$is an integer, $k>1$ is an integer, and $4\left|\left(p^m-1\right)\right.$ [8].

A special place among them is occupied by the perfect ternary sequences (PTSs) with elements {–1, 0, 1}. These are in fact binary alphabet sequences {–1, 1}, but with zero symbols in some positions. However, there are essential differences. First, their length is not limited from above in comparison with perfect binary sequences. Secondly, these sequences are numerous, and their peak factors converge to one with an increase in their length. Thirdly, the hardware implementation of PTS generators is simple, unlike the other generators of the perfect polyphaser sequences with an alphabet volume greater than three. Finally, PTSs can compensate for an absence of perfect binary sequences in a continuous transmission regime. To this end, Levanon and Freedman proposed changing every other zero in the periodically transmitted PTS with ones and minus ones [9]. The initial PTS is used as a reference sequence in a correlator, and integration time is set to be equal to an even number of sequence periods. In this case the peak factor is equal to one, and the values of correlator side-lobes are zero. As a result, the price for mismatched filtering is energy loss in a receiver, but these losses converge to zero with an increase in the PTS length.

Many papers and books are devoted to the design of PTSs and the study of their properties. Notable examples are Ipatov's monograph [1] devoted to periodical discrete signals with optimal correlation properties (1991) and the handbook [2] on sequence design by Fan and Darnell (1996) in which they considered the PTS families known at the time. During past two decades, numerous new PTS families have been discovered, a number of theorems concerning their existence have been formulated and connections established between them and circulate weighing matrices $CW(N,\,K)$ order of N (which is equal to the sequence length) and the weight of K.

In the light of the foregoing and considering the increased interest in PTSs, this paper presents a brief retrospective review of PTSs and their almost 60 years' history. The work also considers some constructions of the sequence generators.

Perfect ternary sequences (brief review). PTS history dates back about 60 years. In 1960 Tompkins generated PTSs with a length of 18 using the exhaustive search method. [5, 10]. Then in 1967 Chang built PTSs with a length of $N=\left(3^n-1\right)/2$ (N is an odd) based on m-sequences over $\mathrm{GF}(3)$. It is worth mentioning that a similar construction method was proposed by Green and Kelsch [11]. In 1977 Moharir [12] found the necessary conditions for the PTS existence and obtained several new sequences using cyclic difference sets. In 1979 Shedd and Sarwate built the PTS family with a length of p^n-1 ($p\geq 2$ – prime number) [18] based on the property of correlation identity of two sequences (Sarwate и Pursley [13]) valid for couples of m-sequences with three-level cross-correlation (Gold [14], Niho [15], Kasami [16], Helleseth [17]).

Then two PTS systematic constructions with length $\left(p^n-1\right)/\left(p^m-1\right)$, where p is a prime number, $n=mk$, $m>1$ is an integer, and $k>3$ is an odd number invented within an interval of several years. The first construction for $p>2$ was based on p^m-ary m-sequences and obtained by Ipatov in 1979 [19]. The second construction for $p=2$ was obtained by Hoholdt and Justesen. It was based on Zinger difference sets [20]. Then Ipatov, Platonov, Samoilov [21] and Kamaletdinov [22] found other PTSs with the same parameters (peak factor and length) as the PTSs reported in [19]. Later it was shown that the Chang and the Green-Kelsch PTSs [10, 11] are a subset of the Ipatov PTSs for $p = 3$; the Hoholdt-Justesen PTSs [20] correspond to the Shedd-Sarwate PTS for $p = 2$, $m = 1$ and odd *n*; and the Moharir PTSs [12] belong to the Ipatov or the Hoholdt-Justesen. PTSs. More details are shown in [2].

In subsequent years, after the discovery of new perfect binary sequences with two-level cross-correlation (GMW sequences, Kasami power function sequences, Welch-Gong sequences and hyperoval Maschietti sequences [23]), a number of papers devoted to their cross-correlation properties appeared. In this respect papers [24, 25, 26, 27, 28, 29, 30, 31] deserve a mention since they allowed various pairs of binary and non-binary sequences with the three-level cross-correlation to be found, thereby satisfying the condition of the Shedd-Sarwate construction. As a result, this led to the possibility of building more PTSs [30].

In 1986, Games [32] built the family of ternary sequences of a length of $\left(q^n - 1\right)/\left(q - 1\right)$ (where *q* is a prime power) using difference sets and quadrics in projective geometry. Games proved that the Hoholdt-Justesen PTSs are the subset of the construction obtained by him. In 1992 Jackson and Wild [33] showed that the Ipatov PTSs are also the subset of the Games PTSs. However, there was a question whether it was possible to generate PTSs for even *n* using the construction proposed by Games. This problem known as the Waterloo problem was resolved by Arasu, Dillon, Jungnickel, and Pott in 1995 using the relative difference sets and weighing matrices [34, 35]. As a result, it was proven that it is possible to build PTSs only for odd *n* using the constructions proposed by Games.

Since then the Ipatov PTSs have been reinvented many times. Lee [5] showed that these PTSs are a subset of perfect *q*-ary sequences which were found using multiplicative characters over $\mathrm{GF}(p)$. Then in 1996 Schotten and Lüke [7] obtained the same PTSs from *w*-cyclic perfect sequences. The Ipatov PTSs for $q = 3$ have been shown to be a subset of the perfect polyphase sequences with zeros built by Boztas and Parampalli [36].

At the same time perfect ternary arrays were investigated; in particular, circular weighing matrices $CW(N, K)$ with an order of *N* and weight of *K* consisting of elements of the set $\{-1, 0, 1\}$. Investigation of $CW(N, K)$ is of a great importance, since there is a biunique correspondence between them and PTSs with a length of *N* and *K* non-zero elements. Detailed reviews of PTAs and $CW(N, K)$ matrices are contained in papers by Arasu and Dillon [37, 38] In 1990 Antweiler, Bömer, and Lüke proposed a new method for constructing PTAs and PTSs [39]. To this end, they used the Kronecker product of known PTSs and ternary aperiodic perfect arrays. Using this method and a computer search, they obtained a new PTS of a length of 33 with an energy efficiency of 0.76.

At the present time, a variety of theorems of existence and non-existence of $CW(N, K)$ with specified parameters *N* and *K* have been obtained [37, 38]. Using these theorems and a computer search $CW(24, 9)$, $CW(71, 25)$, $CW(87, 49)$, $CW(96, 36)$, and $CW(142, 100)$ were found.

Of special interest are combined PTSs produced using a symbol-by-symbol product of two PTSs with relatively prime periods or product of the perfect binary sequence 111–1 of a length of 4 and a PTS of odd length [1,2]. The method of building of PTSs of length 4*N* proposed by Krengel in 2007 [40] should also be mentioned. According to this method, new PTSs of a length of 4*N* can be constructed by mixing the perfect ternary sequences and ternary sequences with the odd-perfect auto-correlation of odd length *N* and the same number of zeros.

Finally, the most recent construction of PTSs of odd length $N = N_1 N_2$ was produced by Krengel in 2017 [41, 42]. New PTSs are derived from shift sequences of length N_1 corresponding to *m*-sequences of length $p^n - 1$ over $\mathrm{GF}(p)$ and PTSs of odd length N_2 where $p > 2$ is a prime, $n = mk$, $m \ge 1$ is an integer, $k \ge 3$ is an odd, $N_1 = \left(p^{mk} - 1\right)/\left(p^m - 1\right)$, and $2N_2 \left| \left(p^m - 1\right)\right.$. it is worth noting that a number of PTSs formed by this method increases, if extended shift sequences are used obtained by means of difference balance functions with d-form properties [43, 44].

Perfect ternary sequence generators. In [1] devices which generate Ipatov PTSs of a length of $\left(p^{mk} - 1\right)/\left(p^m - 1\right)$, $p \ge 3$ a prime and the Hoholdt-

Justesen's PTSs of a length of $\left(2^{mk}-1\right)/\left(2^{m}-1\right)$ for $m \ge 1$ and odd $k \ge 3$ are described in detail. Moreover, in [1] the Hoholdt-Justesen PTSs are presented with an application of the trace functions and transformations that make their hardware implementation more achievable.

The Ipatov PTSs $\mathbf{g}=\{g_i\}$ of a length of $N=\left(p^{mk}-1\right)/\left(p^{m}-1\right)$ are built in accordance with the expression:

$$g_i=(-1)^i\,\psi\left[\mathrm{Tr}_m^n\left(\alpha^i\right)\right],\ \ 0 \le i < N,$$

where $\psi[\cdot]$ is the multiplicative character of $\mathrm{GF}\left(p^m\right)$;

$$\mathrm{Tr}_m^n(x)=\sum_{i=0}^{n/m-1} x^{p^{im}}$$

is the trace of x element of $\mathrm{GF}\left(p^n\right)$ relative to $\mathrm{GF}\left(p^m\right)$; $n=mk$; α is the primitive element of $\mathrm{GF}\left(p^n\right)$, $m \ge 1$; $k \ge 3$ is an odd. It should be recalled that the double-digit multiplicative character of $\mathrm{GF}(q)$ is an image of the multiplicative group of $\mathrm{GF}^*(q)$ of the main field (i.e. all $q-1$ non-zero field elements) on the set of $\{-1, 1\}$ kind of $\psi(\delta)=(-1)^{\log_\beta \delta}$ where $\delta \in \mathrm{GF}(q)$; $\log_\beta \delta$ is the logarithm of δ to the base β; β is the primitive element of $\mathrm{GF}(q)$. Clearly $\log_\beta \delta$ takes one of the values of the set of integers $\{0, 1, 2, \ldots, q-2\}$. In the case of the Ipatov PTSs $q=p^m$, the multiplicative character is also set to zero for element $\delta=0$.

The block-diagram of the Ipatov PTS generator is shown in the Fig. 1. The PTS generator consists of the generator (1) of q-ary m-sequence of length q^k-1, $q=p^m$, $m \ge 1$, $k \ge 3$ – odd; converter (2) making a conversion of input non-zero element (non-zero element of $\mathrm{GF}(q)$) into the double-digit multiplicative character, the value of which is –1 or 1, setting zero symbol (zero element of $\mathrm{GF}(q)$) to zero. The converter output is connected to the first input of

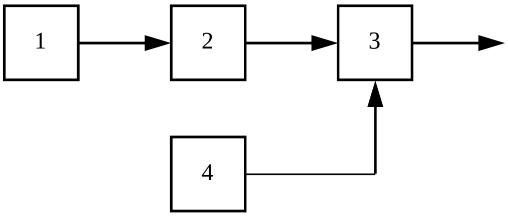

Fig. 1. Block-diagram of Ipatov PTS generator:
1 – q-ary m-sequence generator; 2 – converter; 3 – multiplier;
4 – meander generator.

the multiplier (3), the second input of which is connected to the output of the meander generator (4).

The operating principles and block-diagram of the generator of q-ary m-sequence of length q^k-1 is described in detail in literature (in books by Ipatov [1], Fan and Darnell [2], Golomb and Gong [23], etc.). The converter which calculates the double-digit multiplicative character of the Galois field element can be produced by using various devices. In particular, devices for the direct calculation of the double-digit multiplicative character of any non-zero element of $\mathrm{GF}(q)$ could be used for this purpose [45]. However, such production requires significant resources of hardware and time. Construction of the converter, on the other hand, could be significantly simplified using the preliminary derived table of conversion of non-zero p^m elements x_j into one of the following symbols $\{-1, 0, 1\}$, which is performed using read-only memory (ROM) as proposed in [1].

The Hoholdt-Justesen PTSs $\mathbf{a}=\{a_i\}$ of a length of $N=\left(2^n-1\right)/\left(2^m-1\right)$, $n=mk$, $m \ge 1$, $k \ge 3$ – odd in the interpretation proposed by Ipatov [1] is given by:

$$a_i=\begin{cases} 0, & \mathrm{Tr}_m^n\left(\xi^i\right)=0; \\ e\left[d_i\left(\mathrm{Tr}_m^n\,\xi^i\right)^{q-3}\right], & \mathrm{Tr}_m^n\left(\xi^i\right)\ne 0, \end{cases}$$

where $e(\delta)=(-1)^{\mathrm{Tr}_1^m(\lambda\delta)}$ is the additive character of $\mathrm{GF}(q)$ $\left(\lambda, \delta \in \mathrm{GF}(q)\right)$; $0 \le i < N$, $q=2^m$,

$$d_i=\begin{cases} \sum\limits_{t=1}^{(k-1)/4} \mathrm{Tr}_m^n\left\{\xi^{\left[q^{(8t-1)s}+1\right]i}\right\}, & k \equiv 1 \bmod 4; \\ \sum\limits_{t=1}^{(k-3)/4} \mathrm{Tr}_m^n\left\{\xi^{\left[q^{(8t+1)s}+1\right]i}\right\}, & k \equiv 1 \bmod 4; \end{cases}$$

ξ is the primitive element $\mathrm{GF}\left(q^k\right)$, s is an integer number, and $(s, k)=1$, i. e. the greatest common factor of s and k is equal one.

Fig. 2 shows the block-diagram of the Hoholdt–Justesen PTS generator obtained in accordance with the expressions presented. The generator consists of two generators of linear sequences, (1) – the generator which forms the m-sequence $\left\{\mathrm{Tr}_m^n\left(\xi^i\right)\right\}$ and (2) – the generator which forms the sequence $\{d_i\}$, and (3) – the converter which in the case of $\left\{\mathrm{Tr}_m^n\left(\xi^i\right)\right\}\neq 0$ raises the element $\mathrm{Tr}_m^n\left(\xi^i\right)$ to the power of $q-3$, then multiplies it by the d_i element formed in the generator (2), and calculates the additive character of the multiplication result. In the opposite case, the symbol of zero is formed at the converter output (3). The converter may be obtained by means of ROM for the reasons of simplification [1].

The generation of PTSs [41] which include the Ipatov PTSs can now be considered as a special case. In [42] the generator of PTSs of odd length N_1N_2 where $N_1=\left(p^{mk}-1\right)/\left(p^m-1\right)$; $N_2=\left(p^m-1\right)/(2h)$ is a length of some known PTS, $p>2$ is a prime number; $n=mk$; $m\geq 1$, $h\geq 1$ are integers; $k\geq 3$ is an odd. The above-mentioned method of PTSs building is described in detail in [41].

Let $\mathbf{d}=\{d_i\}$ be a p-ary m-sequence of length p^n-1 with elements

$$d_i=\mathrm{Tr}_1^n\left(\alpha^i\right)=\sum_{j=0}^{n-1}\alpha^{ip^j},\quad n=mk,\ 0\leq i<p^n-1$$

and $\mathbf{b}=\{b_i\}$ be a q-ary m-sequence of length p^n-1, $q=p^m$ with elements

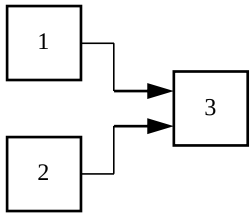

Fig. 2. Block-diagram of Hoholdt–Justesen PTS generator:
1 – generator of the m-sequence $\left\{\mathrm{Tr}_m^n\left(\xi^i\right)\right\}$;
2 – generator of the sequence $\{d_i\}$; 3 – converter

$$b_i=\mathrm{Tr}_m^n\left(\alpha^i\right)=\sum_{j=0}^{n/m-1}\alpha^{ip^{mj}},\quad n=mk,\ 0\leq i<p^n-1,$$

where α is a primitive element of $\mathrm{GF}\left(p^n\right)$.

Let us consider the decomposition array of sequence $\mathbf{d}$ consisting of $T=\left(p^n-1\right)/\left(p^m-1\right)$ rows and p^m-1 columns. Rows in this array are sequences of all zeros or cyclic shifts of some short p-ary m-sequence of length p^m-1. The values of these shifts are defined by the shift sequence $\mathbf{e}$ [23, 24] given as:

$$\mathbf{e}=\{e_i\}=\begin{cases}\infty, & \mathrm{Tr}_m^n\left(\alpha^i\right)=0,\\ \log_\beta\left[\mathrm{Tr}_m^n\left(\alpha^i\right)\right], & \mathrm{Tr}_m^n\left(\alpha^i\right)\neq 0,\end{cases}$$

where $0\leq i<T$, and symbol ∞ points to zero row.

The algorithm of the PTS $\mathbf{v}$ construction consists of four steps:

1. Choose for some PTS $\mathbf{a}$ of odd length N_2 parameters $p\geq 3$, prime; $m\geq 1$, integer; $k\geq 3$, odd, $(2N_2)\left|\left(p^m-1\right)\right.$ and a primitive polynomial of power $n=km$ over $\mathrm{GF}(p)$.

2. Calculate the shift sequence $\mathbf{e}$ of length $N_1=\left(p^n-1\right)/\left(p^m-1\right)$ corresponding to p-ary m-sequence $\mathbf{d}$ of length p^n-1.

3. Construct the array $\mathbf{V}$ of order $N_1\times N_2$ where i-th row is defined by

$$\mathrm{str}_i=\begin{cases}(-1)^{(i+e_i)\bmod 2}\,L^{e_i \bmod N_2}(\mathbf{a}), & e_i\neq\infty,\\ 0K\ 0, & e_i=\infty,\end{cases}$$
$$0\leq i<N_1,$$

and $L^s(\mathbf{a})$ is the operator of the cyclic shift of the sequence $\mathbf{a}$ to the left by s digits.

4. Unfold the array V by columns onto the PTS $\mathbf{v}$ of length N_1N_2.

The analysis shows that if $\left(N_1, N_2\right)=1$, then the lengths of the PTSs $\mathbf{v}$ are equal to lengths of combined PTSs. However, in the case of $\left(N_1, N_2\right)\neq 1$ lengths of the obtained PTSs $\mathbf{v}$ could be unique. Coincidence with the length of the Ipatov sequences is possible when PTS $\mathbf{a}$ is an Ipatov sequence of length

$N_2 = \left(p^{ef} - 1\right) / \left(p^f - 1\right)$ where $e \ge 3$ is an odd number, $f \ge 1$ is an integer, $m = ef$. Note that in this case only one of the PTSs **v** of length $N = \left(p^n - 1\right) / \left(p^f - 1\right)$ thus obtained coincides with the Ipatov PTS. In all other cases PTSs **v** differ from known PTSs, i.e. they are new. In this context, it should be noted that if the sequence $\mathbf{a} = \{1\}$, then the sequence **v** coincides with the Ipatov PTS with length N_1. The peak factor of these sequences is equal to multiplication of the peak factors of PTSs of lengths N_1 and N_2. Since $N_1 \gg N_2$, then the peak factor (and therefore energy losses) of the new sequence will be determined by the peak factor of PTS of length N_2.

In order to generate the periodic PTSs we will proceed as follows. We produce $\hat{\mathbf{a}}$ sequence of length $p^m - 1$ using $\left(p^m - 1\right) / N_2$ periods of the sequence **a**. Then, using expression $b_{i+T} = \beta b_i$, the ternary sequence $\mathbf{v}' = \{v_i'\}$, $0 \le i < p^{mk} - 1$ produced by using $\left(p^m - 1\right) / N_2$ periods of the PTS **v** is described as

$$v_i' = \begin{cases} (-1)^{i+z_i} \, \hat{a}_{z_i}, & b_i \ne 0; \\ 0, & b_i = 0; \end{cases} \quad 0 \le i < p^{mk} - 1,$$

where $z_i = \log_\beta b_i$, $b_i \ne 0$ and $z_i = e_i$ for $0 \le i < T$.

Assuming

$$f\left(b_i\right) = \begin{cases} (-1)^{z_i} \, \hat{a}_{z_i}, & b_i \ne 0; \\ 0, & b_i = 0; \end{cases} \quad 0 \le i < p^{mk} - 1,$$

we finally obtain $v_i' = (-1)^i f\left(b_i\right)$.

The calculation of $f\left(b_i\right)$ could be simplified if, instead of the logarithmic table, we use a table which assigns a double-digit number to the symbol $b_i \in \mathrm{GF}\left(p^m\right)$. It has the value 10 if $f\left(b_i\right) = 1$, the value 01 if $f\left(b_i\right) = -1$, and the value 00 if $f\left(b_i\right) = 0$.

The element $\mathbf{c} \in \mathrm{GF}(q)$, $q = p^m$ could be represented as the sum

$$c_{m-1}\beta^{m-1} + c_{m-2}\beta^{m-1} + \mathrm{K} + c_0,$$

where $c_i \in \mathrm{GF}(p)$ and β is the primitive element of $\mathrm{GF}\left(p^m\right)$. Therefore, for any element **c** of $\mathrm{GF}\left(p^m\right)$ we could assign the *m*-digit *p*-ary number representing as $\left(c_{m-1}, c_{m-2}, \mathrm{K}, c_0\right)$. Written in binary this number consists of $\lceil m \log_2(p) \rceil$ digits and equals $\left(c_{m-1}p^{m-1} + c_{m-2}p^{m-1} + \mathrm{K} + c_0\right)_2$. With this in mind, the mapping table could be produced on the basis of a programmable read-only memory (PROM) with the volume $p^m \times 2$, which uses binary form of the element **c** of $\mathrm{GF}\left(p^m\right)$ as an address input. As a result, the converter block will consist of a series-connected address builder. This will transform *m*-digit *p*-ary representation of elements of $\mathrm{GF}(q)$ on the output of the generator of the *q*-ary m-sequence into $\lceil m \log_2(p) \rceil$-digit binary numbers, the PROM with volume $q \times 2$, and the code converter of the double-digit binary number into the symbol of ternary code $\{-1, 0, 1\}$. It should be noted that if $q = p$, then the address for the PROM is the value of the symbol **c**. In this case the address builder is not required.

Fig. 3 shows the block-diagram for the perfect ternary sequences generator of length $N_1 N_2$. The generator contains the series-connected generator 1 of *q*-ary *m*-sequence of length $q^k - 1$, $q = p^m$, $m \ge 1$ and $k \ge 3$ odd, address builder 2, PROM 3, and code converter 4 of 2-digit binary code into the symbol of the ternary code, the multiplier 5, and the meander generator 6. The output of the code converter is connected to the first input of the multiplier 5; the mean-

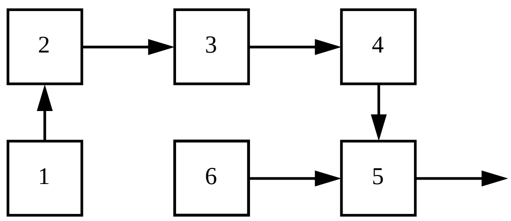

Fig. 3. Block-diagram of PTS generator of the length $N_1 N_2$:
1 – generator of the m-sequence with the length $q^k - 1$; 2 – address builder; 3 – PROM; 4 – code converter; 5 – multiplier;
6 – meander generator.

der generator 6 is connected to the second input of the multiplier.

The combined sequences generator of length N_1N_2 consists of two generators of PTSs with a coprime lengths N_1 and N_2 whose outputs are connected to the inputs of the multiplier. A block-diagram of the quadruple-length PTS generator of looks more sophisticated [40]. In this case, PTSs with length 4*N* are built on the base of interleaving of two sequences: the sequence consisting of two periods of the PTS sequence of odd length *N* and the almost perfect ternary sequence of length 2*N* having twice larger number of zeros in comparison with the PTS with length *N*.

The generator will be constructed in accordance with the following: 1) – an almost perfect ternary sequence is a concatenation of an odd-perfect ternary sequence of length *N* and its inversion; and 2) – the result of multiplication of elements of an odd-perfect sequence of odd length by the alternating sequence $(-1)^i$ is the perfect sequence with the same length [40], [41]. The generator of PTSs of length 4*N* (Fig. 4) consists of 1 – the alternating sequence generator of; 2, 3 – the generators of PTSs with odd length *N* and clock frequency f_T; 4 – the multiplier; 5 – the multiplexer which joins together two input sequences into one output sequence. As a result, the PTS of length 4*N* and double frequency $2f_T$ is produced on the output of the multiplexer.

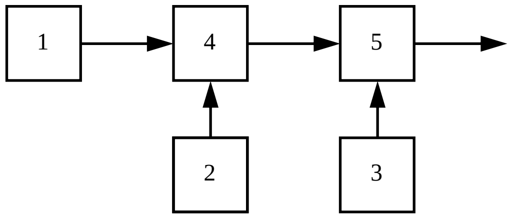

Fig. 4. Block-diagram of PTS with the length $4N$: 1 – meander generator; 2, 3 – generators of the PTPs with odd length *N* and clock frequency f_T; 4 – multiplier; 5 – multiplexer

Conclusion. The study presents a brief retrospective review of 60 years of history of PTSs and considers the generators of some PTS families. Over the past two decades various new PTS families have been constructed and investigated, but the last review of PTSs was published in 1996, leading to the need for this study. The interest in PTSs is generated by the fact that they demonstrate perfect auto-correlation properties and their energy efficiency converges to one with an increase of length. This allows them to be used in modern radio communications and radar systems and in CW and LPI radars in particular.

In addition to informational and bibliographical objectives, this review shows the interconnection between PTSs obtained at different time and their equivalence to the circular weighing matrices.

This review could be useful for developers of various systems where perfect ternary sequences are used.

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