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DESIGN OF BAND FILTERS WITH NON-EQUIRIPPLE AMPLITUDE-FREQUENCY RESPONSES

Abstract.
Introduction. *The calculation of band filter elements by converting low-pass filter (LPF) parameters forms the basis for prototype band filter design. However, the conversion causes problems in cases when the calculated values of circuit elements (resistors and capacitors) do not conform to standard values determined by the GOST standard. Obviously, frequency characteristics of band- filters are distorted when replacing the calculated values of circuit elements by the standard values. To solve this problem, the number of circuit elements with values diverting from the standard can be reduced to zero by solving an additional system of equations that connects parameters of designed and newly-introduced non-equiripple amplitude-frequency responses (AFR).*
Objective. *The objective of this work is to develop a method for calculating band ladder filters having circuit element values corresponding to the published standard.*
Materials and methods. *The filter design process includes two stages. The first of these is to perform a calculation of the prototype polynomial LPF parameters. The calculated parameters are determined as the solution to a system of equations. The equations are formed by equating coefficients at the same powers of variable in expressions of realisable transfer function (TF) and designed filter TF. Initial characteristics are the filter order and frequency response unevenness. A transition to the standard values of circuit elements can be achieved when solving another system of equations that connects LPF converted parameters with unknown parameters of the introduced non-equiripple AFR.*
Results. *TFs of LPF prototypes up to the fifth order and AFRs of band-pass filters (BPF) and band-rejection filters up to the tenth order are presented. Analytical expressions of non-equiripple and equiripple ARF are used to estimate distortions of the latter when a BPF centre frequency is tuned by using variable inductors or capacitors. The integral quadratic function of a variable is taken as a measure of real frequency response distortions. The tenth order BPF calculation example is given.*
Conclusion. *The presented band filter calculation methods and provided example demonstrate the feasibility of a filter design method based on the solution of non-linear equation systems. In contrast to approximation methods of ideal filter frequency response using special functions and tabular filters design, the presented method supports high-order filter calculation for any initial requirements without using reference data.*

Key words: transfer function, low-pass filter, frequency transformation, band-pass filter, band-rejection filter, tunable filter

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СИНТЕЗ ПОЛОСНЫХ ФИЛЬТРОВ С НЕРАВНОВОЛНОВЫМИ АМПЛИТУДНО-ЧАСТОТНЫМИ ХАРАКТЕРИСТИКАМИ

Аннотация.
Введение. *При расчете полосных фильтров элементы цепи могут быть определены посредством преобразования параметров фильтра нижних частот (ФНЧ), являющегося прототипом синтезируемого фильтра. Проблема может возникнуть в случае, если в результате преобразования номиналы синтезированных элементов (резисторов и конденсаторов) выпадают из шкал значений, определенных межгосударственным стандартом. Очевидно, что при замене расчетных значений стандартными частотные характеристики полосных фильтров искажаются. Число компонентов, расчетные номиналы которых не соответствуют стандартному ряду, может быть сведено к нулю решением дополнительной системы уравнений, связывающей параметры синтезированной и вновь вводимой неравноволновой амплитудно-частотных характеристик (АЧХ).*
Цель работы. *Разработка методики расчета полосных фильтров лестничной структуры с элементами, соответствующими стандартным значениям.*
Материалы и методы. *Процесс синтеза включает 2 этапа. На первом этапе рассчитываются параметры полиномиального ФНЧ-прототипа. Расчетные параметры определяются в результате решения системы уравнений, образованных приравниванием коэффициентов при одинаковых степенях переменной в выражениях реализуемой передаточной функции (ПФ) и ПФ синтезируемого фильтра. Исходными характеристиками являются порядок фильтра и неравномерность передачи цепи. Переход к номинальным значениям всех элементов выполнен при решении еще одной системы уравнений, связывающих преобразованные параметры ФНЧ с неизвестными (искомыми) параметрами вновь вводимой неравноволновой АЧХ.*
Результаты. *Представлены ПФ ФНЧ-прототипов до пятого порядка и АЧХ полосно-пропускающих фильтров (ППФ) и полосно-заграждающих фильтров до десятого порядка. Аналитические выражения неравноволновой и равноволновой АЧХ применены для оценки искажений последней при изменении центральной частоты настройки полосных фильтров с помощью переменных индуктивностей или конденсаторов. В качестве меры искажений реальной частотной характеристики принята интегральная квадратичная функция переменной величины. Приведен пример расчета ППФ десятого порядка.*
Заключение. *Представленные методики расчета полосных фильтров и приведенный пример наглядно демонстрируют возможности метода синтеза фильтров, основанного на решении систем нелинейных уравнений. В отличие от методов аппроксимации идеальной характеристики фильтра в частотной области с помощью специальных функций и табличного проектирования фильтров рассмотренный метод позволяет рассчитать фильтр высокого порядка для любых исходных требований, не прибегая к справочным данным.*

Ключевые слова: передаточная функция, фильтр нижних частот, преобразование частоты, полосно-пропускающий фильтр, полосно-заграждающий фильтр, перестраиваемый фильтр

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Источник финансирования. Инициативная работа.

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Introduction. It is known [1]–[3] that a band-pass filter (BPF) or band-rejection filter (BRF) design can be reduced to a low-pass filter (LPF) design with certain parameters. The latter task is solved by various methods (e.g. [4], [5]). To design a LPF with a filter order higher than two given cutoff frequency values and resistive elements at input and output of a quadrupole, a design method based on tabular parameters of so-called normalised filters is widely used [6], [7]. In [8], a method for determining circuit parameters based on solving a system of non-linear equations is investigated in detail. The equations are formed by equating coefficients at the same powers of variable in expressions of realisable transfer function

(TF) and designed filter TF. The conversion of LPF parameters, consisting in a designed band-pass filter prototype, is carried out using a frequency conversion method. The essence of the method is the copying of electric circuit characteristics at a certain scale over an entire axis of a variable ω into a positive semi-axis of a new variable ω' with corresponding circuit and LPF parameter changes. The conversion causes problems if the calculated of circuit element (resistors and capacitors) values are at variance with the standard values (with series designations E6, E12, E24, etc.) determined by the GOST standard [9]. Obviously, the frequency characteristics of band filters are distorted when replacing the calculated values of circuit elements by the standard ones. However, the number of circuit elements with values different from standard can be reduced to zero by solving an additional system of equations that connects parameters of designed and newly introduced non-equiripple amplitude-frequency responses – hereinafter referred to as frequency response.

The objective of the present work is to develop a calculation method of band-pass ladder filters with circuit element values corresponding to standard ones (hereinafter referred to as standard elements).

Fig. 1, 2 show LPF schematic diagrams composed of Г-, Т- and П-sections. Symbols on the schematic diagrams are as follows: $\dot{U}_{\text{in}}$, and $\dot{U}_{\text{out}}$ are complex amplitudes of input and output voltages, respectively; *r* is the active resistance, including the internal resistance of the source; C_i, L_k, $i, k = \overline{1, n}$ are capacitance and inductance in transverse and longitudinal branches, respectively (*n* is a filter order); *R* is the load resistance, and K_{a} is the amplifier gain.

In general, the TF of a filter is

$$H^{(n)}(p) = \dot{U}_{\text{out}}(p) / \dot{U}_{\text{in}}(p), \quad p = \sigma + j\omega.$$

In schematic diagrams (Fig. 1, 2) TFs of filters are rational functions of a variable $s = j\omega$, where the numerator is a constant value, while the denominator is a polynomial of degree *n* (polynomial LPF). Dividing the rational function numerator and denominator over ω_{c}^n, where ω_{c} is an angular cutoff frequency, allows an expression of $H^{(n)}(s_{\text{N}})$ to be obtained as a function of a normalised imaginary frequency $s_{\text{N}} = j\,\omega/\omega_{\text{c}} = j\omega_{\text{N}}$.

Calculation methods of band-pass filters. The conversion of LPF to BPF with a centre frequency $\omega_0 = \omega_{\text{c}}$ is carried out by a variable replacement [2]:

$$s_{\text{N}} \to \Theta\left(s'_{\text{N}} + 1/s'_{\text{N}}\right), \qquad (1)$$

where Θ is a positive number; $s'_{\text{N}} = j\omega'_{\text{N}}$ is an imaginary part of the converted normalised complex frequency, where $\omega'_{\text{N}} = \omega/\omega_0$ is an angular frequency, which is normalised relative to the centre frequency ω_0. Values of the variable ω'_{N}, which correspond to the normalised frequency value ω_{N}, are defined as equation roots

$$\omega'^2_{\text{N}} - (\omega_{\text{N}}/\Theta)\omega'_{\text{N}} - 1 = 0:$$

$$\omega'_{\text{N}1,2} = \sqrt{1 + \omega_{\text{N}}^2/\left(4\Theta^2\right)} \mp \omega_{\text{N}}/(2\Theta); \omega_{\text{N}} \ge 0.$$

The geometric mean of frequencies $\omega'_{\text{N}1,2}$ gives a normalised centre frequency of BPF: $\sqrt{\omega'_{\text{N}1}\omega'_{\text{N}2}} = 1$. The frequency difference is $\omega'_{\text{N}2} - \omega'_{\text{N}1} = \omega_{\text{N}}/\Theta$, thus $\Theta = \omega_{\text{N}}/\left(\omega'_{\text{N}2} - \omega'_{\text{N}1}\right)$. If $\omega_{\text{N}} = 1$, then Θ is defined as a reciprocal of the difference between the normalised BPF frequencies obtained by converting the LPF cutoff frequency. Thus, Θ is a BPF selectivity factor.

BPF frequency response values $H_{\text{BP}}^{(2n)}(\omega_{\text{N}})$ at points $\omega'_{\text{N}1,2}$ are equal to the LPF prototype frequency response values $H^{(n)}(\omega_{\text{N}})$ at the corresponding point ω_{N}. For example, BPF frequency response values when converting from a Chebyshev LPF $H_{\text{BP C}}^{(2n)}(\omega_{\text{N}})$ at corner frequencies $\omega'_{\text{N cf}} = \sqrt{1 + 1/\left(4\Theta^2\right)} \mp 1/(2\Theta)$ are equal to $1/\sqrt{1+\varepsilon^2}$, where ε is a constant value, and only with $\varepsilon = 1$, which corresponds to 3 dB frequency response, unevenness $H_{\text{BP C}}^{(2n)}\left(\omega'_{\text{N cf}}\right) = 1/\sqrt{2}$.

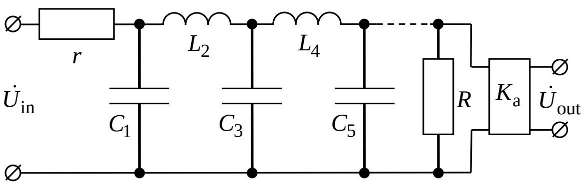

Fig. 1. Schematic diagram of LPF with the capacitor in the transverse branch at the input

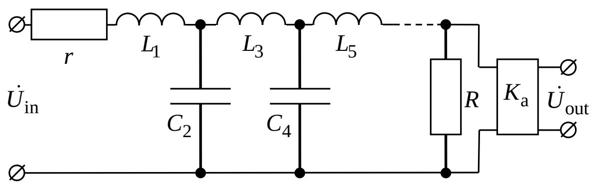

Fig. 2. Schematic diagram of LPF with the inductor in the longitudinal branch at the input

If $H^{(n)}(1) = 1/\sqrt{2}$, then

$$H_{\text{BP}}^{(2n)}\left(\sqrt{1+\frac{1}{4\Theta^2}}-\frac{1}{2\Theta}\right)=$$
$$=H_{\text{BP}}^{(2n)}\left(\sqrt{1+\frac{1}{4\Theta^2}}+\frac{1}{2\Theta}\right)=\frac{1}{\sqrt{2}},$$

and $\Theta = 1/\left(\omega'_{\text{N2}} - \omega'_{\text{N1}}\right)$ in the case of the BPF quality factor Q, which is defined as a reciprocal of a normalised BPF bandwidth at a frequency response level of $1/\sqrt{2}$.

Conversion to angular frequencies transforms equation (1) into the following expression:

$$j\frac{\omega}{\omega_{\text{c}}} \to \Theta\left[j\frac{\omega}{\omega_0}+1\Big/\left(j\frac{\omega}{\omega_0}\right)\right]. \tag{2}$$

Multiplying both left and right sides of the equation (2) by $\omega_0 C$ gives:

$$j\omega C \to j\omega\Theta C + \omega_0^2\Theta C/(j\omega). \tag{3}$$

Terms ωC and $1/(\omega L)$ are capacitive and inductive conductivities, respectively; thus, from equation (3) it follows that when conversion (1) takes place, then capacitance C is replaced by a parallel resonant circuit with parameters $C_{\text{pr}} = \Theta C$ and $L_{\text{pr}} = 1/\left(\omega_0^2\Theta C\right)$. Replacement of C with L in equation (3) confirms that when conversion to BPF takes place, then inductance L is replaced by a series resonant circuit with parameters $L_{\text{sr}} = \Theta L$, $C_{\text{sr}} = 1/\left(\omega_0^2\Theta L\right)$. The obtained circuits are tuned to a frequency of ω_0. The resistor values r and R and amplifier gain K_{a} remain unchanged.

Fig. 3, 4 show schematic diagrams of the $2n^{\text{th}}$-order BPF with elements $C_{\text{pr}i}$, $L_{\text{pr}i}$, $C_{\text{sr}k}$, $L_{\text{sr}k}$, $i, k = \overline{1, n}$, obtained by Fig. 1, 2 sections conversions, respectively.

Considering conversions performed, LPF prototype capacitances and inductances are connected with BPF circuit capacitances as follows:

$$C_i = C_{\text{pr}i}/\Theta;\ \ L_k = 1/\left(\omega_0^2\Theta C_{\text{sr}k}\right).$$

Table 1 shows the TF of LPF prototypes of BPF with $n = \overline{1, 5}$ filter orders with the capacitor in the transverse branch at the input (Fig. 1) $H_{\text{b}\,C}^{(n)}(s_{\text{N}})$ and with the inductor in the longitudinal branch at the input (Fig. 2) $H_{\text{b}\,L}^{(n)}(s_{\text{N}})$ having elements $C_{\text{pr}i}$, $C_{\text{sr}k}$, and the parameter Θ as coefficients of the variable s_{N}. To express the TF of LPF by means of the circuit elements shown in Fig. 1, 2, it is necessary to use substitutions $C_{\text{pr}i} = \Theta C_i$, $C_{\text{sr}k} = 1/\left(\omega_0^2\Theta L_k\right)$, replacing ω_0 by ω_{c}.

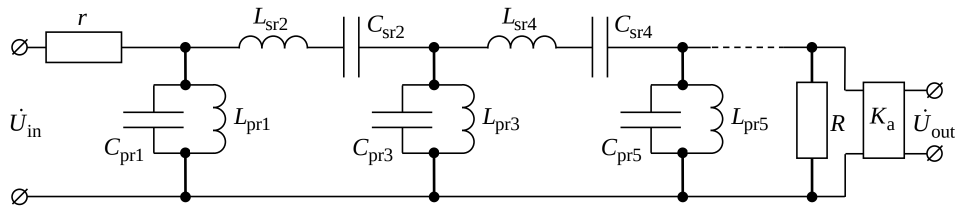

Fig 3. Schematic diagram of the 2nth-order BPF with the parallel resonant circuit in the transverse branch at the input

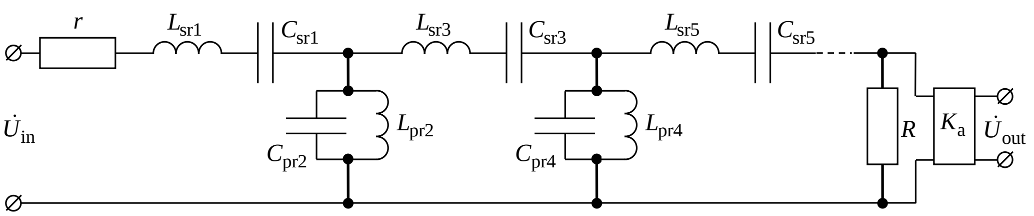

Fig 4. Schematic diagram of the 2nth-order BPF with the series resonant circuit in the longitudinal branch at the input

Table 1. Transfer functions of LPF prototypes of BPF

$n=1$
$H_{\text{b}\,C}^{(1)}(s_{\text{N}})=\dfrac{K_{\text{a}}\Theta}{\omega_0 rC_{\text{pr1}}}\Big/\left[s_{\text{N}}+\dfrac{(r+R)\Theta}{\omega_0 rC_{\text{pr1}}R}\right];\quad H_{\text{b}\,L}^{(1)}(s_{\text{N}})=K_{\text{a}}\omega_0 RC_{\text{sr1}}\Theta\big/\left[s_{\text{N}}+\omega_0(r+R)C_{\text{sr1}}\Theta\right]$
$n=2$
$H_{\text{b}\,C}^{(2)}(s_{\text{N}})=\dfrac{K_{\text{a}}RC_{\text{sr2}}\Theta^2}{rC_{\text{pr1}}}\Big/\left[s_{\text{N}}^2+\dfrac{1+\omega_0^2 rC_{\text{pr1}}RC_{\text{sr2}}}{\omega_0 rC_{\text{pr1}}}\Theta s_{\text{N}}+\dfrac{(r+R)C_{\text{sr2}}}{rC_{\text{pr1}}}\Theta^2\right];$ $H_{\text{b}\,L}^{(2)}(s_{\text{N}})=\dfrac{K_{\text{a}}C_{\text{sr1}}\Theta^2}{C_{\text{pr2}}}\Big/\left[s_{\text{N}}^2+\dfrac{1+\omega_0^2 rC_{\text{sr1}}RC_{\text{pr2}}}{\omega_0 RC_{\text{pr2}}}\Theta s_{\text{N}}+\dfrac{(r+R)C_{\text{sr1}}}{RC_{\text{pr2}}}\Theta^2\right]$
$n=3$
$H_{\text{b}\,C}^{(3)}(s_{\text{N}})=\dfrac{K_{\text{a}}C_{\text{sr2}}\Theta^3\big/\left(\omega_0 rC_{\text{pr1}}C_{\text{pr3}}\right)}{s_{\text{N}}^3+\dfrac{rC_{\text{pr1}}+RC_{\text{pr3}}}{\omega_0 rC_{\text{pr1}}RC_{\text{pr3}}}\Theta s_{\text{N}}^2+\dfrac{1+\omega_0^2 r\left(C_{\text{pr1}}+C_{\text{pr3}}\right)RC_{\text{sr2}}}{\omega_0^2\, rC_{\text{pr1}}RC_{\text{pr3}}}\Theta^2 s_{\text{N}}+\dfrac{(r+R)C_{\text{sr2}}}{\omega_0 rC_{\text{pr1}}RC_{\text{pr3}}}\Theta^3};$ $H_{\text{b}\,L}^{(3)}(s_{\text{N}})=\dfrac{K_{\text{a}}\omega_0 RC_{\text{sr1}}C_{\text{sr3}}\Theta^3\big/C_{\text{pr2}}}{s_{\text{N}}^3+\omega_0\left(rC_{\text{sr1}}+RC_{\text{sr3}}\right)\Theta s_{\text{N}}^2+\dfrac{C_{\text{sr1}}+C_{\text{sr3}}\left(1+\omega_0^2 rC_{\text{sr1}}RC_{\text{pr2}}\right)}{C_{\text{pr2}}}\Theta^2 s_{\text{N}}+\dfrac{\omega_0(r+R)C_{\text{sr1}}C_{\text{sr3}}}{C_{\text{pr2}}}\Theta^3}$
$n=4$
$H_{\text{b}\,C}^{(4)}(s_{\text{N}})=\left[K_{\text{a}}RC_{\text{sr2}}C_{\text{sr4}}\Theta^4\big/\left(rC_{\text{pr1}}C_{\text{pr3}}\right)\right]\big/\Lambda_{\text{b}\,C}^{(4)},$ where $\Lambda_{\text{b}\,C}^{(4)}=s_{\text{N}}^4+\dfrac{1+\omega_0^2 rC_{\text{pr1}}RC_{\text{sr4}}}{\omega_0 rC_{\text{pr1}}}\Theta s_{\text{N}}^3+\dfrac{r\left(C_{\text{pr1}}+C_{\text{pr3}}\right)C_{\text{sr2}}+\left(rC_{\text{pr1}}+RC_{\text{pr3}}\right)C_{\text{sr4}}}{rC_{\text{pr1}}C_{\text{pr3}}}\Theta^2 s_{\text{N}}^2+$ $+\dfrac{C_{\text{sr2}}+C_{\text{sr4}}+\omega_0^2 r\left(C_{\text{pr1}}+C_{\text{pr3}}\right)RC_{\text{sr2}}C_{\text{sr4}}}{\omega_0 rC_{\text{pr1}}C_{\text{pr3}}}\Theta^3 s_{\text{N}}+\dfrac{(r+R)C_{\text{sr2}}C_{\text{sr4}}}{rC_{\text{pr1}}C_{\text{pr3}}}\Theta^4;$ $H_{\text{b}\,L}^{(4)}(s_{\text{N}})=\left[K_{\text{a}}C_{\text{sr1}}C_{\text{sr3}}\Theta^4\big/\left(C_{\text{pr2}}C_{\text{pr4}}\right)\right]\big/\Lambda_{\text{b}\,L}^{(4)},$ where $\Lambda_{\text{b}\,L}^{(4)}=s_{\text{N}}^4+\dfrac{1+\omega_0^2 rC_{\text{sr1}}RC_{\text{pr4}}}{\omega_0 RC_{\text{pr4}}}\Theta s_{\text{N}}^3+\dfrac{\left(rC_{\text{sr1}}+RC_{\text{sr3}}\right)C_{\text{pr2}}+R\left(C_{\text{sr1}}+C_{\text{sr3}}\right)C_{\text{pr4}}}{RC_{\text{pr2}}C_{\text{pr4}}}\Theta^2 s_{\text{N}}^2+$ $+\dfrac{C_{\text{sr1}}+C_{\text{sr3}}+\omega_0^2 rC_{\text{sr1}}R\left(C_{\text{pr2}}+C_{\text{pr4}}\right)C_{\text{sr3}}}{\omega_0 RC_{\text{pr2}}C_{\text{pr4}}}\Theta^3 s_{\text{N}}+\dfrac{(r+R)C_{\text{sr1}}C_{\text{sr3}}}{RC_{\text{pr2}}C_{\text{pr4}}}\Theta^4$
$n=5$
$H_{\text{b}\,C}^{(5)}(s_{\text{N}})=\left[K_{\text{a}}C_{\text{sr2}}C_{\text{sr4}}\Theta^5\big/\left(\omega_0 rC_{\text{pr1}}C_{\text{pr3}}C_{\text{pr5}}\right)\right]\big/\Lambda_{\text{b}\,C}^{(5)},$ where $\Lambda_{\text{b}\,C}^{(5)}=s_{\text{N}}^5+\dfrac{rC_{\text{pr1}}+RC_{\text{pr5}}}{\omega_0 rC_{\text{pr1}}RC_{\text{pr5}}}\Theta s_{\text{N}}^4+\left(\dfrac{C_{\text{pr1}}C_{\text{sr4}}+C_{\text{sr2}}C_{\text{pr5}}}{C_{\text{pr1}}C_{\text{pr5}}}+\dfrac{C_{\text{sr2}}+C_{\text{sr4}}}{C_{\text{pr3}}}+\dfrac{1}{\omega_0^2 rC_{\text{pr1}}RC_{\text{pr5}}}\right)\Theta^2 s_{\text{N}}^3+$ $+\dfrac{\left(rC_{\text{pr1}}+RC_{\text{pr5}}\right)\left(C_{\text{sr2}}+C_{\text{sr4}}\right)+\left(rC_{\text{sr2}}+RC_{\text{sr4}}\right)C_{\text{pr3}}}{\omega_0 rC_{\text{pr1}}RC_{\text{pr3}}C_{\text{pr5}}}\Theta^3 s_{\text{N}}^2+\dfrac{C_{\text{sr2}}+C_{\text{sr4}}+\omega_0^2 r\left(C_{\text{pr1}}+C_{\text{pr3}}+C_{\text{pr5}}\right)RC_{\text{sr2}}C_{\text{sr4}}}{\omega_0^2 rC_{\text{pr1}}RC_{\text{pr3}}C_{\text{pr5}}}\Theta^4 s_{\text{N}}+$ $+\dfrac{(r+R)C_{\text{sr2}}C_{\text{sr4}}}{\omega_0 rC_{\text{pr1}}RC_{\text{pr3}}C_{\text{pr5}}}\Theta^5;$ $H_{\text{b}\,L}^{(5)}(s_{\text{N}})=\left[K_{\text{a}}\omega_0 RC_{\text{sr1}}C_{\text{sr3}}C_{\text{sr5}}\Theta^5\big/\left(C_{\text{pr2}}C_{\text{pr4}}\right)\right]\big/\Lambda_{\text{b}\,L}^{(5)},$ where $\Lambda_{\text{b}\,L}^{(5)}=s_{\text{N}}^5+\omega_0\left(rC_{\text{sr1}}+RC_{\text{sr5}}\right)\Theta s_{\text{N}}^4+\left(\dfrac{C_{\text{sr1}}+C_{\text{sr3}}}{C_{\text{pr2}}}+\dfrac{C_{\text{sr3}}+C_{\text{sr5}}}{C_{\text{pr4}}}+\omega_0^2 rC_{\text{sr1}}RC_{\text{sr5}}\right)\Theta^2 s_{\text{N}}^3+$

End of the table 1

$+\omega_0\left[C_{sr1}\left(\frac{r}{C_{pr4}}+\frac{R}{C_{pr2}}\right)C_{sr5}+\frac{(C_{pr2}+C_{pr4})C_{sr3}}{C_{pr2}C_{pr4}}(rC_{sr1}+RC_{sr5})\right]\Theta^3 s_N^2+$
$+\frac{C_{sr1}(C_{sr3}+C_{sr5})+C_{sr3}C_{sr5}\left[1+\omega_0^2 rC_{sr1}R(C_{pr2}+C_{pr4})\right]}{C_{pr2}C_{pr4}}\Theta^4 s_N+\frac{\omega_0(r+R)C_{sr1}C_{sr3}C_{sr5}}{C_{pr2}C_{pr4}}\Theta^5$

The designed TF of the n[th]-order LPF with an equiripple frequency response in the bandwidth is defined as [*]:

$$\tilde{H}_{LP}^{(n)}(s_N)=\tilde{K}\Big/\left(s_N^n+\tilde{b}_{n-1}s_N^{n-1}+\cdots+\tilde{b}_1 s_N+\tilde{b}_0\right), \quad (4)$$

where coefficients $\tilde{K}$, $\tilde{b}_i$ are real positive numbers. The frequency response of the polynomial LPF with coefficients $\tilde{K}$, $\tilde{b}_i$ evenly approximates the ideal frequency response in the passband and monotonically decreases in the stopband. The methodology and calculations examples of the coefficients $\tilde{K}$, $\tilde{b}_{n-1}$, $\tilde{b}_{n-2}$, …, $\tilde{b}_0$, as well as frequencies of the frequency response extrema $\tilde{\omega}_{Ni}$, $i=\overline{2,\ n}$ for various n and $\tilde{\delta}$, are given in [10]. The boundary condition imposed on the frequency response is $\tilde{H}_{LP}^{(n)}(1)=1/\sqrt{2}$. By equating numerators and coefficients of variables raised to the same powers s_N^{n-1}, s_N^{n-2}, ..., s_N^0 in denominators of the converted TF $H^{(n)}(s_N)$ (Table 1) and $\tilde{H}_{LP}^{(n)}(s_N)$ (4), the system of $n+1$ equations is obtained, which allows the parameters of the BPF K_a, r, C_{pri}, C_{srk}, R to be determined with the given frequency response unevenness $\tilde{\delta}$ and quality factor Q. The total number of unknowns is $n+3$, with $n+2$ standard elements (resistors and capacitors); thus, the nominal values of the two elements are set arbitrarily from the selected series.

Table 1 shows that if $s_N=0$, then the TF value of the LPF prototype – and, consequently, the value of its frequency response at zero frequency – is defined as:

$$H^{(n)}(0)=K_a R/(r+R). \quad (5)$$

On the other side, $\tilde{H}_{LP}^{(n)}(0)=\tilde{K}/\tilde{b}_0$ [see (4)], hence $K_a=\tilde{K}(r+R)/(\tilde{b}_0 R)$. This can be used as means of verifying the solution to the system of equations.

The transition to the standard values of other elements can be done when solving another system of equations that connects LPF converted parameters with unknown parameters of newly introduced non-equiripple frequency response $\widehat{H}^{(n)}(\omega_N)$ [**] with q extrema, where q can take one of the possible integer values over the section $1\le q\le n$.

The unevennesses of $\widehat{H}^{(n)}(\omega_N)$ at the extrema $\widehat{\omega}_{Ni}$, $i=\overline{1,\ q}$, can be defined as:

$$\widehat{\delta}_i=20\lg\widehat{H}^{(n)}(\widehat{\omega}_{Ni}). \quad (6)$$

Since $\widehat{\omega}_{N1}=0$, taking into account (5),

$$\lg\widehat{H}^{(n)}(0)=\lg\left[K_a R/(r+R)\right].$$

The introduced definition of the frequency response unevenness differs from the definition of the equiripple frequency response unevenness that is given in [10]:

$$\tilde{\delta}=20\left|\lg\left\{\tilde{H}^{(n)}(0)\Big/\left[2-\tilde{H}^{(n)}(0)\right]\right\}\right|,$$

which characterises the equiripple frequency response at the section.

The system of $2q+1$ equations used to find the $n+2$ standard elements of BPF, corrected amplifier gain K_a and $2q$ parameters of non-equiripple frequency response $\widehat{H}^{(n)}(\omega_N)$: $\widehat{\delta}_i$, $\widehat{\omega}_{N2}$, $\widehat{\omega}_{N3}$, ..., $\widehat{\omega}_{Nq}$, $\widehat{Q}$ is defined as:

$$\begin{cases}20\lg\left[K_a R/(r+R)\right]=\widehat{\delta}_1;\\ 20\lg\widehat{H}^{(n)}(\widehat{\omega}_{Ni})=\widehat{\delta}_i,\ i=\overline{2,\ q};\\ \widehat{H}^{(n)}(1)=1/\sqrt{2};\\ d\widehat{H}^{(n)}(\widehat{\omega}_{Ni})\big/d\,\widehat{\omega}_{Ni}=0,\ i=\overline{2,\ q};\\ \int_0^1\frac{\partial}{\partial K_a}\left[\tilde{H}_{LP}^{(n)}(\omega_N)-\widehat{H}^{(n)}(\omega_N)\right]^2 d\,\omega_N=0,\end{cases} \quad (7)$$

where $\tilde{H}_{LP}^{(n)}(\omega_N)$ is the frequency response of the polynomial nth-order LPF. For $n=1$

$$\tilde{H}_{LP}^{(1)}(\omega_N)=\tilde{K}\Big/\sqrt{\omega_N^2+\tilde{b}_0^2}; \quad (8)$$

for $n\ge 2$

[*] The sign "~" indicates that the parameter belongs to the equiripple frequency response.

[**] The sign ⌢ indicates that the parameter belongs to the non-equiripple frequency response.

$$\tilde{H}_{\text{LP}}^{(n)}(\omega_{\text{N}}) = \tilde{K} \big/ \sqrt{\tilde{P}}, \tag{9}$$

where for even n

$$\tilde{P} = \left[\omega_{\text{N}}^{n} - \sum_{j=0}^{(n/2)-1} (-1)^{\frac{n-2}{2}-j} \tilde{b}_{2j}\, \omega_{\text{N}}^{2j}\right]^2 + \left[\sum_{j=1}^{n/2} (-1)^{\frac{n}{2}-j} \tilde{b}_{2j-1}\, \omega_{\text{N}}^{2j-1}\right]^2;$$

and for odd $n \ge 3$

$$\tilde{P} = \left[\omega_{\text{N}}^{n} - \sum_{j=1}^{(n-1)/2} (-1)^{\frac{n-1}{2}-j} \tilde{b}_{2j-1} \omega_{\text{N}}^{2j-1}\right]^2 + \left[\sum_{j=0}^{(n-1)/2} (-1)^{\frac{n-1}{2}-j} \tilde{b}_{2j}\, \omega_{\text{N}}^{2j}\right]^2.$$

Frequency response $\hat{H}^{(n)}(\omega_{\text{N}})$ (see (7)) is determined by $\tilde{H}_{\text{LP}}^{(n)}(\omega_{\text{N}})$ (8), (9) with replacement of $\tilde{K}$ by an expression of the TF numerator of LPF prototypes $H_{\text{b}\,C}^{(n)}(s_{\text{N}})$ or $H_{\text{b}\,L}^{(n)}(s_{\text{N}})$ (see Table 1). The coefficients $\tilde{b}_{2j}$, $\tilde{b}_{2j-1}$ in the expressions of $\tilde{P}$ are substituted by the coefficients of the variables s_{N}^{2j}, s_{N}^{2j-1} in the denominators of the same equations.

It should be noted that the introduced definition of the frequency response unevennesses $\hat{H}^{(n)}(\hat{\omega}_{\text{N}i})$ (6) as the frequency response values in decibels at the extreme points does not guarantee the passband ripple oscillations around the unit value. This can be seen from equation (6) expressed as: $\hat{\delta}_i = 20\lg\left[\hat{H}^{(n)}(\hat{\omega}_{\text{N}i})/1\right]$. Multiplication of the numerator and denominator of the fraction under the logarithm sign by the same number is not changing the logarithm value. To bind the ripple mean value to the unit value, the integral equation (the last one in the system) is introduced into the equation system (7), which provides the least quadratic deviation of the non-equiripple frequency response $\hat{H}^{(n)}(\omega_{\text{N}})$ from the designed frequency response $\tilde{H}_{\text{LP}}^{(n)}(\omega_{\text{N}})$ by varying the amplifier gain K_{a}.

The total number of unknowns is $(n+2q+3)$, while the number of equations is $(2q+1)$. The difference $(n+2)$ is equal to the number of BPF standard elements. When forming the equation system, the $(n+2)$ element values of BPF r, $C_{\text{pr}i}$, $C_{\text{sr}k}$, R, which are calculated for the implementation of the equiripple frequency response, are substituted by the closest possible standard values, while the initial parameters $\tilde{\delta}$, $\tilde{\omega}_{\text{N}2}$, $\tilde{\omega}_{\text{N}3}$, .., $\tilde{\omega}_{\text{N}n}$, Q and the calculated value of K_{a} are used as an initial approximation of the desired solution of the system of equations.

Transition to the standard values of the circuit elements causes distortions of the resulting frequency response. In addition to the ripple value changes, their number may also decrease. The number of extrema required to form the equations system (7) is determined from the BPF frequency responses graphs $\hat{H}_{\text{BP}}^{(2n)}(\omega_{\text{N}})$ with the parameters K_{a} and standard values of the filter elements as $q = (p-1)/2 + 1$, where p is a number of extrema of the function $\hat{H}_{\text{BP}}^{(2n)}(\omega_{\text{N}})$.

The frequency response expressions of BPF with filter orders of $2n = 2$, 4, 6, 8, 10 with parallel $\hat{H}_{\text{BP}_{\text{pr}}C}^{(2n)}(\omega_{\text{N}})$ and series $\hat{H}_{\text{BP}_{\text{sr}}C}^{(2n)}(\omega_{\text{N}})$ circuits at the inputs according to the schematic diagrams in Fig. 3 and Fig. 4, respectively, are presented in Table 2. Taking into account the correlations

$$L_{\text{pr}i}C_{\text{pr}i} = L_{\text{sr}k}C_{\text{sr}k} = 1/\omega_0^2,$$

inductances are expressed by means of the corresponding circuits capacitances; thus, in the equations (Table 2) only symbols $C_{\text{pr}i}$ and $C_{\text{sr}k}$ appear (this is signified by a subscript C).

Possible negative values of $\hat{\delta}_i$, obtained as a solution of the system (7), correspond to the frequency response values $\hat{H}^{(n)}(\hat{\omega}_{\text{N}i}) < 1$.

The analytical expressions of $H_{\text{BP}}^{(2n)}(\omega_{\text{N}})$ can be used to estimate the frequency response distortions when changing the BPF centre frequency. As a measure of the BPF frequency response $\hat{H}_{\text{BP}}^{(2n)}(\omega_{\text{N}})$ distortions when an offset from the centre frequency is equal to $\Delta\omega_0$ can be taken as the definite integral value of the square of the difference between the functions $\tilde{H}_{\text{BP}}^{(2n)}(\omega_{\text{N}})$ and $\hat{H}_{\text{BP}}^{(2n)}\left[\omega_{\text{N}}, (1+\Delta)\omega_0\right]$ over the section

Table 2. BPF frequency responses

$2n = 2$

$$H^{(2)}_{\mathrm{BP_{pr}}C}(\omega_\mathrm{N}) = \frac{\left[K_\mathrm{a}/(\omega_0 rC_\mathrm{pr1})\right]\omega_\mathrm{N}}{\sqrt{\left(\omega_\mathrm{N}^2-1\right)^2+\left(\dfrac{r+R}{\omega_0 rC_\mathrm{pr1}R}\omega_\mathrm{N}\right)^2}};\quad H^{(2)}_{\mathrm{BP_{sr}}C}(\omega_\mathrm{N}) = \frac{K_\mathrm{a}\omega_0 RC_\mathrm{sr1}\omega_\mathrm{N}}{\sqrt{\left(\omega_\mathrm{N}^2-1\right)^2+\left[\omega_0(r+R)C_\mathrm{sr1}\omega_\mathrm{N}\right]^2}}$$

$2n = 4$

$$H^{(4)}_{\mathrm{BP_{pr}}C}(\omega_\text{н}) = \frac{K_\mathrm{a}RC_\mathrm{sr2}}{rC_\mathrm{pr1}}\omega_\mathrm{N}^2 \Bigg/ \sqrt{\left\{\omega_\mathrm{N}^4-\left[2+\frac{(r+R)C_\mathrm{sr2}}{rC_\mathrm{pr1}}\right]\omega_\mathrm{N}^2+1\right\}^2+\left[\frac{1+\omega_0^2 rC_\mathrm{pr1}RC_\mathrm{sr2}}{\omega_0 rC_\mathrm{pr1}}\left(\omega_\mathrm{N}^3-\omega_\mathrm{N}\right)\right]^2};$$

$$H^{(4)}_{\mathrm{BP_{sr}}C}(\omega_\mathrm{N}) = \frac{K_\mathrm{a}C_\mathrm{sr1}}{C_\mathrm{pr2}}\omega_\mathrm{N}^2 \Bigg/ \sqrt{\left\{\omega_\mathrm{N}^4-\left[2+\frac{(r+R)C_\mathrm{sr1}}{RC_\mathrm{pr2}}\right]\omega_\mathrm{N}^2+1\right\}^2+\left[\frac{1+\omega_0^2 rC_\mathrm{sr1}RC_\mathrm{pr2}}{\omega_0 RC_\mathrm{pr2}}\left(\omega_\mathrm{N}^3-\omega_\mathrm{N}\right)\right]^2}$$

$2n = 6$

$$H^{(6)}_{\mathrm{BP_{pr}}C} = \left[K_\mathrm{a}C_\mathrm{sr2}\omega_\mathrm{N}^3\Big/\left(\omega_0 rC_\mathrm{pr1}C_\mathrm{pr3}\right)\right]\Big/\sqrt{\Lambda^{(6)}_{\mathrm{BP_{pr}}C}},$$

where

$$\Lambda^{(6)}_{\mathrm{BP_{pr}}C} = \left\{\omega_\mathrm{N}^6-\left[3+\frac{1+\omega_0^2 r\left(C_\mathrm{pr1}+C_\mathrm{pr3}\right)RC_\mathrm{sr2}}{\omega_0^2 rC_\mathrm{pr1}RC_\mathrm{pr3}}\right]\left(\omega_\mathrm{N}^4-\omega_\mathrm{N}^2\right)-1\right\}^2+$$
$$+\left[\frac{rC_\mathrm{pr1}+RC_\mathrm{pr3}}{\omega_0 rC_\mathrm{pr1}RC_\mathrm{pr3}}\left(\omega_\mathrm{N}^5+\omega_\mathrm{N}\right)-\frac{2\left(rC_\mathrm{pr1}+RC_\mathrm{pr3}\right)+(r+R)C_\mathrm{sr2}}{\omega_0 rC_\mathrm{pr1}RC_\mathrm{pr3}}\omega_\mathrm{N}^3\right]^2;$$

$$H^{(6)}_{\mathrm{BP_{sr}}C}(\omega_\mathrm{N}) = \left(K_\mathrm{a}\omega_0 RC_\mathrm{sr1}C_\mathrm{sr3}\omega_\mathrm{N}^3\Big/C_\mathrm{pr2}\right)\Big/\sqrt{\Lambda^{(6)}_{\mathrm{BP_{sr}}C}},$$

where

$$\Lambda^{(6)}_{\mathrm{BP_{sr}}C} = \left\{\omega_\mathrm{N}^6-\left[3+\frac{C_\mathrm{sr}\left(1+\omega_0^2 rC_\mathrm{pr2}RC_\mathrm{sr3}\right)+C_\mathrm{sr3}}{C_\mathrm{pr2}}\right]\left(\omega_\mathrm{N}^4-\omega_\mathrm{N}^2\right)-1\right\}^2+$$
$$+\left(\omega_0 rR\right)^2\left\{\left(\frac{C_\mathrm{sr1}}{R}+\frac{C_\mathrm{sr3}}{r}\right)\left(\omega_\mathrm{N}^5+\omega_\mathrm{N}\right)-\left[2\left(\frac{C_\mathrm{sr1}}{R}+\frac{C_\mathrm{sr3}}{r}\right)+\frac{(r+R)C_\mathrm{sr1}C_\mathrm{sr3}}{rC_\mathrm{pr2}R}\right]\omega_\mathrm{N}^3\right\}^2$$

$2n = 8$

$$H^{(8)}{}_{\mathrm{BP_{pr}}C}(\omega_\mathrm{N}) = \left[K_\mathrm{a}RC_\mathrm{sr2}C_\mathrm{sr4}\omega_\mathrm{N}^4\Big/\left(rC_\mathrm{pr1}C_\mathrm{pr3}\right)\right]\Big/\sqrt{\Lambda^{(8)}_{\mathrm{BP_{pr}}C}},$$

where

$$\Lambda^{(8)}_{\mathrm{BP_{pr}}C} = \left\{\omega_\mathrm{N}^8-\left(4+\frac{C_\mathrm{sr2}+C_\mathrm{sr4}}{C_\mathrm{pr3}}+\frac{rC_\mathrm{sr2}+RC_\mathrm{sr4}}{rC_\mathrm{pr1}}\right)\left(\omega_\mathrm{N}^6+\omega_\mathrm{N}^2\right)+\right.$$
$$\left.+\left[6+\frac{C_\mathrm{sr2}C_\mathrm{sr4}}{C_\mathrm{pr1}C_\mathrm{pr3}}+2\left(\frac{C_\mathrm{sr2}}{C_\mathrm{pr1}}+\frac{C_\mathrm{sr2}+C_\mathrm{sr4}}{C_\mathrm{pr3}}\right)+\left(2+\frac{C_\mathrm{sr2}}{C_\mathrm{pr3}}\right)\frac{RC_\mathrm{sr4}}{rC_\mathrm{pr1}}\right]\omega_\mathrm{N}^4+1\right\}^2+$$
$$+\left\{\frac{1+\omega_0^2 rC_\mathrm{pr1}RC_\mathrm{sr4}}{\omega_0 rC_\mathrm{pr1}}\left(\omega_\mathrm{N}^7-\omega_\mathrm{N}\right)-\left[\left(3+\frac{C_\mathrm{sr2}}{C_\mathrm{pr1}}+\frac{C_\mathrm{sr2}}{C_\mathrm{pr3}}\right)\omega_0 RC_\mathrm{sr4}+\frac{C_\mathrm{sr2}+3C_\mathrm{pr3}+C_\mathrm{sr4}}{\omega_0 rC_\mathrm{pr1}C_\mathrm{pr3}}\right]\left(\omega_\mathrm{N}^5-\omega_\mathrm{N}^3\right)\right\}^2;$$

$$H^{(8)}_{\mathrm{BP_{sr}}C}(\omega_\mathrm{N}) = \left[K_\mathrm{a}C_\mathrm{sr1}C_\mathrm{sr3}\omega_\mathrm{N}^4\Big/\left(C_\mathrm{pr2}C_\mathrm{pr4}\right)\right]\Big/\sqrt{\Lambda^{(8)}_{\mathrm{BP_{sr}}C}},$$

where

$$\Lambda^{(8)}_{\mathrm{BP_{sr}}C} = \left\{\omega_\mathrm{N}^8-\left(4+\frac{C_\mathrm{sr1}+C_\mathrm{sr3}}{C_\mathrm{pr2}}+\frac{rC_\mathrm{sr1}+RC_\mathrm{sr3}}{RC_\mathrm{pr4}}\right)\left(\omega_\mathrm{N}^6+\omega_\mathrm{N}^2\right)+\right.$$
$$\left.+\left[6+\frac{C_\mathrm{sr1}C_\mathrm{sr3}}{C_\mathrm{pr2}C_\mathrm{pr4}}+2\left(\frac{C_\mathrm{sr3}}{C_\mathrm{pr4}}+\frac{C_\mathrm{sr1}+C_\mathrm{sr3}}{C_\mathrm{pr2}}\right)+\left(2+\frac{C_\mathrm{sr3}}{C_\mathrm{pr2}}\right)\frac{rC_\mathrm{sr1}}{RC_\mathrm{pr4}}\right]\omega_\mathrm{N}^4+1\right\}^2+$$
$$+\left\{\frac{1+\omega_0^2 rC_\mathrm{sr1}RC_\mathrm{pr4}}{\omega_0 RC_\mathrm{pr4}}\left(\omega_\mathrm{N}^7-\omega_\mathrm{N}\right)-\left[\left(3+\frac{C_\mathrm{sr3}}{C_\mathrm{pr2}}+\frac{C_\mathrm{sr3}}{C_\mathrm{pr4}}\right)\omega_0 rC_\mathrm{sr1}+\frac{C_\mathrm{sr1}+3C_\mathrm{pr2}+C_\mathrm{sr3}}{\omega_0 RC_\mathrm{pr2}C_\mathrm{pr4}}\right]\left(\omega_\mathrm{N}^5-\omega_\mathrm{N}^3\right)\right\}^2$$

End of Table 2

$2n=10$
$H^{(10)}_{\mathrm{BP_{pr}}C}(\omega_\mathrm{N})=\left[K_\mathrm{a}C_\mathrm{sr2}C_\mathrm{sr4}\omega_\mathrm{N}^5\big/\left(\omega_0 rC_\mathrm{pr1}C_\mathrm{pr3}C_\mathrm{pr5}\right)\right]\Big/\sqrt{\Lambda^{(10)}_{\mathrm{BP_{pr}}C}},$
where
$\Lambda^{(10)}_{\mathrm{BP_{pr}}C}=\Bigg\{\omega_\mathrm{N}^{10}-\left(5+\frac{C_\mathrm{sr2}}{C_\mathrm{pr1}}+\frac{C_\mathrm{sr2}+C_\mathrm{sr4}}{C_\mathrm{pr3}}+\frac{C_\mathrm{sr4}}{C_\mathrm{pr5}}+\frac{1}{\omega_0^2 rC_\mathrm{pr1}RC_\mathrm{pr5}}\right)\left(\omega_\mathrm{N}^8-\omega_\mathrm{N}^2\right)+$
$+\left[10+3\left(\frac{C_\mathrm{sr2}}{C_\mathrm{pr1}}+\frac{C_\mathrm{sr2}+C_\mathrm{sr4}}{C_\mathrm{pr3}}+\frac{C_\mathrm{sr4}}{C_\mathrm{pr5}}\right)+\frac{C_\mathrm{pr1}+C_\mathrm{pr3}+C_\mathrm{pr5}}{C_\mathrm{pr1}C_\mathrm{pr3}C_\mathrm{pr5}}C_\mathrm{sr2}C_\mathrm{sr4}+\frac{C_\mathrm{sr2}+3C_\mathrm{pr3}+C_\mathrm{sr4}}{\omega_0^2 rC_\mathrm{pr1}RC_\mathrm{pr3}C_\mathrm{pr5}}\right]\left(\omega_\mathrm{N}^6-\omega_\mathrm{N}^4\right)-1\Bigg\}^2+$
$+\Bigg[\frac{rC_\mathrm{pr1}+RC_\mathrm{pr5}}{\omega_0 rC_\mathrm{pr1}RC_\mathrm{pr5}}\left(\omega_\mathrm{N}^9+\omega_\mathrm{N}\right)-\frac{\left(rC_\mathrm{pr1}+RC_\mathrm{pr5}\right)\left(C_\mathrm{sr2}+4C_\mathrm{pr3}+C_\mathrm{sr4}\right)+\left(rC_\mathrm{sr2}+RC_\mathrm{sr4}\right)C_\mathrm{pr3}}{\omega_0 rC_\mathrm{pr1}RC_\mathrm{pr3}C_\mathrm{pr5}}\left(\omega_\mathrm{N}^7+\omega_\mathrm{N}^3\right)+$
$+\frac{2\left(rC_\mathrm{pr1}+RC_\mathrm{pr5}\right)\left(C_\mathrm{sr2}+3C_\mathrm{pr3}+C_\mathrm{sr4}\right)+2\left(rC_\mathrm{sr2}+RC_\mathrm{sr4}\right)C_\mathrm{pr3}+(r+R)C_\mathrm{sr2}C_\mathrm{sr4}}{\omega_0 rC_\mathrm{pr1}RC_\mathrm{pr3}C_\mathrm{pr5}}\omega_\mathrm{N}^5\Bigg]^2;$
$H^{(10)}_{\mathrm{BP_{sr}}C}(\omega_\mathrm{N})=\left[K_\mathrm{a}\omega_0 RC_\mathrm{sr1}C_\mathrm{sr3}C_\mathrm{sr5}\omega_\mathrm{N}^5\big/\left(C_\mathrm{pr2}C_\mathrm{pr4}\right)\right]\Big/\sqrt{\Lambda^{(10)}_{\mathrm{BP_{sr}}C}},$
where
$\Lambda^{(10)}_{\mathrm{BP_{sr}}C}=\Bigg\{\omega_\mathrm{N}^{10}-\left(5+\frac{C_\mathrm{sr1}+C_\mathrm{sr3}}{C_\mathrm{pr2}}+\frac{C_\mathrm{sr3}+C_\mathrm{sr5}}{C_\mathrm{pr4}}+\omega_0^2 rC_\mathrm{sr1}RC_\mathrm{sr5}\right)\left(\omega_\mathrm{N}^8-\omega_\mathrm{N}^2\right)+\Bigg[10+3\left(\frac{C_\mathrm{sr1}+C_\mathrm{sr3}}{C_\mathrm{pr2}}+\frac{C_\mathrm{sr3}+C_\mathrm{sr5}}{C_\mathrm{pr4}}\right)+$
$+\frac{C_\mathrm{sr1}\left(C_\mathrm{sr3}+C_\mathrm{sr5}\right)+C_\mathrm{sr3}C_\mathrm{sr5}}{C_\mathrm{pr2}C_\mathrm{pr4}}+\omega_0^2\left(3+\frac{C_\mathrm{sr3}}{C_\mathrm{pr2}}+\frac{C_\mathrm{sr3}}{C_\mathrm{pr4}}\right)rC_\mathrm{sr1}RC_\mathrm{sr5}\Bigg]\left(\omega_\mathrm{N}^6-\omega_\mathrm{N}^4\right)-1\Bigg\}^2+$
$+\left(\omega_0 rR\right)^2\Bigg\{\left(\frac{C_\mathrm{sr1}}{R}+\frac{C_\mathrm{sr5}}{r}\right)\left(\omega_\mathrm{N}^9+\omega_\mathrm{N}\right)-\left[\left(4+\frac{C_\mathrm{sr3}}{C_\mathrm{pr2}}+\frac{C_\mathrm{sr3}+C_\mathrm{sr5}}{C_\mathrm{pr4}}\right)\frac{C_\mathrm{sr1}}{R}+\left(4+\frac{C_\mathrm{sr1}+C_\mathrm{sr3}}{C_\mathrm{pr2}}+\frac{C_\mathrm{sr3}}{C_\mathrm{pr4}}\right)\frac{C_\mathrm{sr5}}{r}\right]\left(\omega_\mathrm{N}^7+\omega_\mathrm{N}^3\right)+$
$+2\left[\left(3+\frac{C_\mathrm{sr3}}{C_\mathrm{pr2}}+\frac{C_\mathrm{sr3}+C_\mathrm{sr5}}{C_\mathrm{pr4}}+\frac{C_\mathrm{sr3}C_\mathrm{sr5}}{2C_\mathrm{pr2}C_\mathrm{pr4}}\right)\frac{C_\mathrm{sr1}}{R}+\left(3+\frac{C_\mathrm{sr1}+C_\mathrm{sr3}}{C_\mathrm{pr2}}+\frac{C_\mathrm{sr3}}{C_\mathrm{pr4}}+\frac{C_\mathrm{sr1}C_\mathrm{sr3}}{2C_\mathrm{pr2}C_\mathrm{pr4}}\right)\frac{C_\mathrm{sr5}}{r}\right]\omega_\mathrm{N}^5\Bigg\}^2$

$$\left[\sqrt{1-1\big/\left(4Q^2\right)}-1\big/(2Q),\ \sqrt{1-1\big/\left(4Q^2\right)}+1\big/(2Q)\right]:$$

$$I^{(2n)}(\Delta)=\int\limits_{\sqrt{1-1/\left(4Q^2\right)}-1/(2Q)}^{\sqrt{1-1/\left(4Q^2\right)}+1/(2Q)}\Big\{\tilde{H}^{(2n)}_\mathrm{BP}(\omega_\mathrm{N})-\hat{H}^{(2n)}_\mathrm{BP}\left[\omega_\mathrm{N},\ (1+\Delta)\omega_0\right]\Big\}^2 d\,\omega_\mathrm{N},$$

where $\tilde{H}^{(2n)}_\mathrm{BP}(\omega_\mathrm{N})$ is an absolute value of TF of the 2*n*th-order BPF, which is obtained by conversion (1) of TF $\tilde{H}^{(n)}_\mathrm{LP}(s_\mathrm{N})$ (4); $\hat{H}^{(2n)}_\mathrm{BP}\left[\omega_\mathrm{N},(1+\Delta)\omega_0\right]$ is a function of the BPF normalised frequency ω_N and centre frequency $(1+\Delta)\omega_0$, $-1<\Delta$. Analytical expressions of $\tilde{H}^{(2n)}_\mathrm{BP}(\omega_\mathrm{N})$ for $2n=2,\ 4, 6, 8, 10$ are given in [10].

The numerical estimation of the frequency response distortions in case of retuning BPF is determined by an average value of the function $I^{(2n)}(\Delta)$ over the section $\left[\Delta_\mathrm{l}\le\Delta\le\Delta_\mathrm{u}\right]$:

$$s_\mathrm{av}=\frac{1}{\Delta_\mathrm{u}-\Delta_\mathrm{l}}\int\limits_{\Delta_\mathrm{l}}^{\Delta_\mathrm{u}} I^{(2n)}(\Delta)\,d\Delta.$$

A number of works is dedicated to the problem of retuning electric filters (see [11]–[13]). By analogy with [13], where the retuning coefficient is introduced as a ratio of centre frequencies in the final and initial states, the retuning coefficient χ is defined as a ratio of upper $(1+\Delta_\mathrm{u})\omega_0$ and lower $(1+\Delta_\mathrm{l})\omega_0$ BPF centre frequencies with acceptable integral frequency response distortions $I^{(2n)}(\Delta_\mathrm{u})$ and $I^{(2n)}(\Delta_\mathrm{l})$, respectively:

$$\chi=(1+\Delta_\mathrm{u})\omega_0\big/\left[(1+\Delta_\mathrm{l})\omega_0\right]=(1+\Delta_\mathrm{u})\big/(1+\Delta_\mathrm{l}).$$

Considering that the amplifier gain K_a remains unchanged during the retuning process, parameters $\hat{\delta}_2$, $\hat{\delta}_3$, ..., $\hat{\delta}_q$, $\hat{\omega}_{\mathrm{N}2}$, $\hat{\omega}_{\mathrm{N}3}$, ..., $\hat{\omega}_{\mathrm{N}q}$, $\hat{Q}$ of LPF prototype frequency responses $\hat{H}^{(n)}\left[\omega_\mathrm{N},\ (1+\Delta_\mathrm{l})\omega_0\right]$ and $\hat{H}^{(n)}\left[\omega_\mathrm{N},(1+\Delta_\mathrm{u})\omega_0\right]$ are solutions of the corresponding systems of $2q-1$ equations, which are formed by excluding the first and last equations of the system (7) and with replacement in expressions for $\widehat{H}^{(n)}(\omega_{\mathrm{N}})$ of the centre frequency of ω_0 by $(1+\Delta)\omega_0$:

$$\begin{cases} 20\lg \widehat{H}^{(n)}\left[\widehat{\omega}_{\mathrm{N}i},(1+\Delta_{\mathrm{l}})\omega_0\right]=\widehat{\delta}_i,\ i=\overline{2,\ q_{\mathrm{l}}}; \\ \widehat{H}^{(n)}\left[1,\ (1+\Delta_{\mathrm{l}})\omega_0\right]=1/\sqrt{2}; \\ d\widehat{H}^{(n)}\left[\widehat{\omega}_{\mathrm{N}i},(1+\Delta_{\mathrm{l}})\omega_0\right]\Big/d\,\widehat{\omega}_{\mathrm{N}i}=0,\ i=\overline{2,\ q_{\mathrm{l}}}; \end{cases} \quad (10)$$

$$\begin{cases} 20\lg \widehat{H}^{(n)}\left[\widehat{\omega}_{\mathrm{N}i},(1+\Delta_{\mathrm{u}})\omega_0\right]=\widehat{\delta}_i,\ i=\overline{2,\ q_{\mathrm{u}}}; \\ \widehat{H}^{(n)}\left[1,\ (1+\Delta_{\mathrm{u}})\omega_0\right]=1/\sqrt{2}; \\ d\widehat{H}^{(n)}\left[\widehat{\omega}_{\mathrm{N}i},(1+\Delta_{\mathrm{u}})\omega_0\right]\Big/d\,\widehat{\omega}_{\mathrm{N}i}=0,\ i=\overline{2,\ q_{\mathrm{u}}}, \end{cases} \quad (11)$$

where q_{l}, q_{u} – are extrema numbers of functions $\widehat{H}^{(n)}\left[\widehat{\omega}_{\mathrm{N}i},(1+\Delta_{\mathrm{l}})\omega_0\right]$, $\widehat{H}^{(n)}\left[\widehat{\omega}_{\mathrm{N}i},(1+\Delta_{\mathrm{u}})\omega_0\right]$ respectively.

The control of the filter centre frequency with fixed standard values of capacitances is carried out by changing circuits inductances, for example, by using variometers. The inductance overlap ratio for each variometer is defined as:

$$k_L=L_{\max}/L_{\min}=(1+\Delta_{\mathrm{u}})^2\Big/(1+\Delta_{\mathrm{l}})^2,$$

where $L_{\max}$, $L_{\min}$ are the maximum (lower limit of the overlapping range) and minimum (upper limit) of the variometer inductances.

In order to estimate the frequency response distortions in case of BPF centre frequency retuning by changing the circuits capacitances, we shall express $H_{\mathrm{BP}}^{(2n)}(\omega_{\mathrm{N}})$ through the circuits inductances $L_{\mathrm{pr}i}$ and $L_{\mathrm{sr}k}$, by substitutions in expressions for $H_{\mathrm{BP_{pr}}C}^{(2n)}(\omega_{\mathrm{N}})$, $H_{\mathrm{BP_{sr}}C}^{(2n)}(\omega_{\mathrm{N}})$ in Table 2 of values below:

$$C_{\mathrm{pr}i}=1\Big/\left(\omega_0^2\,L_{\mathrm{pr}i}\right);\ \ C_{\mathrm{sr}k}=1\Big/\left(\omega_0^2\,L_{\mathrm{sr}k}\right).$$

The initial values of the inductances, which correspond to the centre frequency, are the inductances of the BPF with the equiripple frequency response $\tilde{H}_{\mathrm{BP}}^{(2n)}(\omega_{\mathrm{N}})$. The capacitance overlap ratios for each capacitor are equal and defined as:

$$k_C=C_{\max}/C_{\min}=(1+\Delta_{\mathrm{u}})^2\Big/(1+\Delta_{\mathrm{l}})^2,$$

where $C_{\max}$, $C_{\min}$ are the maximum and minimum circuit capacitances at the centre frequencies of the filter $(1+\Delta_{\mathrm{l}})\omega_0$ and $(1+\Delta_{\mathrm{u}})\omega_0$, respectively.

Thus, the filter design process of the $2n^{\mathrm{th}}$-order BPF having centre frequency ω_0, quality factor Q and amplifier gain K_{a} includes two stages. The first stage is a calculation of the LPF prototype parameters, whose elements C_i and L_k are expressed through the BPF circuit capacitances $C_{\mathrm{pr}i}$ and $C_{\mathrm{sr}k}$, respectively. The calculated parameters are determined by solving the system of equations $(n+2)$ formed by equating the coefficients for equal degrees of variable in the expressions of the PF low-pass filter of the prototype $H_{\mathrm{b}\,C}^{(n)}(s_{\mathrm{N}})$ and the PF low-pass filter with equiripple response $\tilde{H}_{\mathrm{LP}}^{(n)}(s_{\mathrm{H}})$. The initial characteristics are the order of the filter n and the unevenness of transmission of the circuit δ.

The transition to the standard values of circuit elements can be achieved when solving another system of equations that connects LPF converted parameters with unknown parameters of newly introduced non-equiripple frequency response $\widehat{H}^{(n)}(\omega_{\mathrm{N}})$. At the second stage of the filter design, the non-standard BPF elements values are substituted by the closest possible standard values, while the parameters $\tilde{K}_{\mathrm{a}}$, $\tilde{Q}$, $\tilde{\delta}$, $\tilde{\omega}_{\mathrm{N}2}$, $\tilde{\omega}_{\mathrm{N}3}$, ..., $\tilde{\omega}_{\mathrm{N}n}$ of the equiripple frequency response $\tilde{H}_{\mathrm{LP}}^{(n)}(\omega_{\mathrm{N}})$ are used as an initial approximation when calculating LPF prototype non-equiripple frequency response $\widehat{H}^{(n)}(\omega_{\mathrm{N}})$ with parameters $\widehat{K}_{\mathrm{a}}$, $\widehat{Q}$, $\widehat{\delta}_1$, $\widehat{\delta}_2$, ..., $\widehat{\delta}_q$, $\widehat{\omega}_{\mathrm{N}2}$, $\widehat{\omega}_{\mathrm{N}3}$, ..., $\widehat{\omega}_{\mathrm{N}q}$. With conversion of capacitances, the inductance values are defined as:

$$L'_{\mathrm{pr}i}=1\Big/\left(\omega_0^2\,C'_{\mathrm{pr}i}\right),\ \ L'_{\mathrm{sr}k}=1\Big/\left(\omega_0^2\,C'_{\mathrm{sr}k}\right),$$

where $C'_{\mathrm{pr}i}$, $C'_{\mathrm{sr}k}$ are standard capacitance values.

At the final stage, the frequency response of the designed BPF can be corrected by tuning to another centre frequency $\omega_{0\mathrm{cor}}$ by using all filter inductances to achieve the least quadratic deviation of the function $\widehat{H}_{\mathrm{BP}}^{(2n)}\left[\omega_{\mathrm{N}},\ \omega_{0\mathrm{cor}}\right]$ from the equiripple frequency response $\tilde{H}_{\mathrm{BP}}^{(2n)}(\omega_{\mathrm{N}})=\left|\tilde{H}_{\mathrm{BP}}^{(2n)}(s_{\mathrm{N}})\right|$, where $\tilde{H}_{\mathrm{BP}}^{(2n)}(s_{\mathrm{N}})$ is obtained from $\tilde{H}_{\mathrm{LP}}^{(n)}(s_{\mathrm{N}})$ by the normalised frequency s_{N} conversion.

Example. Let us consider calculation of the circuit element parameters and frequency response of a

10th-order BPF with the parallel resonant circuit in the transverse branch at the input (Fig. 3), having circuit centre frequency of $\omega_0 = 10^5$ rad/s and frequency response parameters $\tilde{\delta} = 0.1$, $Q = 10$. The TF coefficients $\tilde{H}_{\text{LP}}^{(5)}(s_{\text{N}})$ of the polynomial 5th-order LPF with the equiripple frequency response unevenness $\tilde{\delta} = 0.1$ and value of $1/\sqrt{2}$ at the cutoff frequency $\omega_{\text{N}} = 1$ are equal to [10]: $\tilde{K} = 0.217744$, $\tilde{b}_4 = 1.535234$, $\tilde{b}_3 = 2.147160$, $\tilde{b}_2 = 1.635204$, $\tilde{b}_1 = 0.862123$, $\tilde{b}_0 = 0.216497$; the normalised extrema frequencies are $\tilde{\omega}_{\text{N}2} = 0.272$, $\tilde{\omega}_{\text{N}3} = 0.517$, $\tilde{\omega}_{\text{N}4} = 0.712$, $\tilde{\omega}_{\text{N}5} = 0.837$. By equating the coefficients of $H_{\text{b}\,C}^{(5)}(s_{\text{N}})$ and $\tilde{H}_{\text{LP}}^{(5)}(s_{\text{N}})$, a system of 6 equations with 8 unknowns is formed.

Assuming that $r = 100\ \Omega$, $R = 51\ \Omega$. , the solution of the equation system is:

$$K_{\text{a}} = 2.978,\ C_{\text{pr1}} = 2.99\ \mu\text{F},\ C_{\text{sr2}} = 10.74\ \text{nF},$$
$$C_{\text{pr3}} = 3.78\ \mu\text{F},\ C_{\text{sr4}} = 12.57\ \text{nF},$$
$$C_{\text{pr5}} = 1.63\ \mu\text{F}.$$

System of equations solution verification:

$$K_{\text{a}} = \tilde{K}(r+R)/\left(\tilde{b}_0 R\right) = 2.978.$$

The closest standard capacitances values from the E12 series to the obtained value C_{pr1} are 2.7 μF and 3.3 μF. For other capacitances, standard values are $C'_{\text{sr2}} = 11$ nF, $C'_{\text{pr3}} = 3.9$ μF, $C'_{\text{sr4}} = 13$ nF, $C'_{\text{pr5}} = 1.6$ μF. For an accurate approximation of the function $\hat{H}_{\text{BP}_{\text{pr}}C}^{(10)}(\omega_{\text{N}})$ to $\tilde{H}_{\text{BP}}^{(10)}(\omega_{\text{N}})$ the capacitor C_{pr1} is replaced in the BPF circuit by two parallel capacitors with nominal capacitances of 1.5 μF, thus obtaining an equivalent capacitance of $C'_{\text{pr1}} = 3$ μF.

Fig. 5, *a* shows the 10th-order BPF frequency response with capacities of $C'_{\text{pr1}} = 2.7,\ 3.0$ и 3.3 μF. The magnified central parts of BPF frequency responses are shown in Fig. 5, *b*. From the figure it can be seen that the number of local extrema of the LPF prototype frequency response is $q = 3$ when $C'_{\text{pr1}} = 2.7$ μF and that it is equal to 5 with other capacitance values. The solutions of the equations system (7) for three C'_{pr1} values and the selected centre frequency ω_0 are given in Table 3 (three left columns). To make a conversion to the normalised

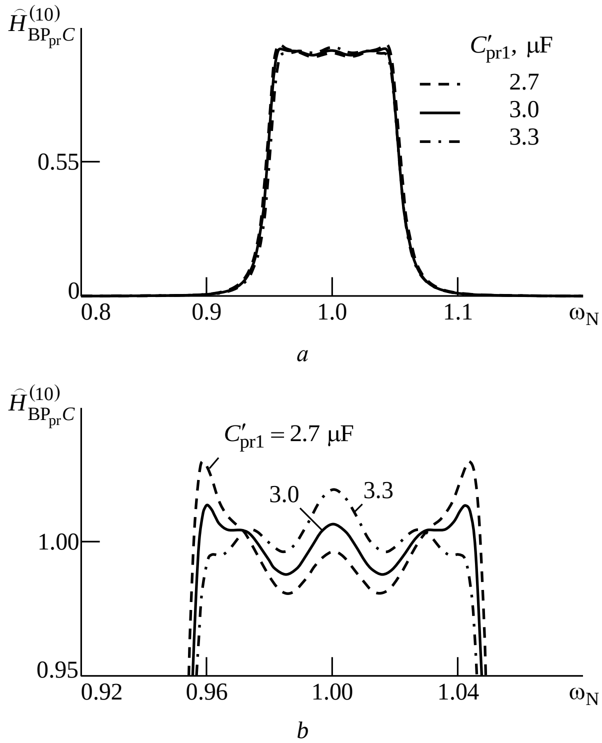

Fig. 5. BPF frequency responses (*a*); central parts of BPF frequency responses (*b*)

extrema frequencies of the BPF frequency response $\hat{H}_{\text{BP}_{\text{pr}}C}^{(10)}(\omega_{\text{N}})$, the following equation is used:

$$\hat{\omega}'_{\text{N}i_{1,\,2}} = \sqrt{1 + \hat{\omega}_{\text{N}i}^2/\left(4\hat{Q}^2\right)} \mp \hat{\omega}_{\text{N}i}/\left(2\hat{Q}\right).$$

It is possible to apply the equations systems (10), (11) to determine the parameters of the BPF

Table 3. LPF prototype frequency response parameters of the 10th-order BPF

Parameter	Tuning to				
	center frequency			extreme frequencies	
ω_0, rad/s	10^5			$0.8 \cdot 10^5$	$1.2 \cdot 10^5$
Δ	0			–0.2	0.2
C'_{pr1}, μF	2.7	3.0	3.3	3.0	3.0
q	3	5	5	4	3
K_{a}	2.950	2.981	3.019	2.981	2.981
$\hat{\delta}_1$	–0.033	0.059	0.169	0.059	0.059
$\hat{\delta}_2$	–0.169	–0.104	–0.031	0.422	–0.786
$\hat{\delta}_3$	0.258	0.041	0.041	–0.031	0.075
$\hat{\delta}_4$	–	0.039	–0.043	0.387	–
$\hat{\delta}_5$	–	0.121	–0.040	–	–
$\hat{Q}$	9.799	9.989	10.175	10.043	10.006
$\hat{\omega}_{\text{N}2}$	0.288	0.305	0.327	0.372	0.350
$\hat{\omega}_{\text{N}3}$	0.828	0.611	0.543	0.675	0.701
$\hat{\omega}_{\text{N}4}$	–	0.662	0.752	0.852	–
$\hat{\omega}_{\text{N}5}$	–	0.821	0.794	–	–

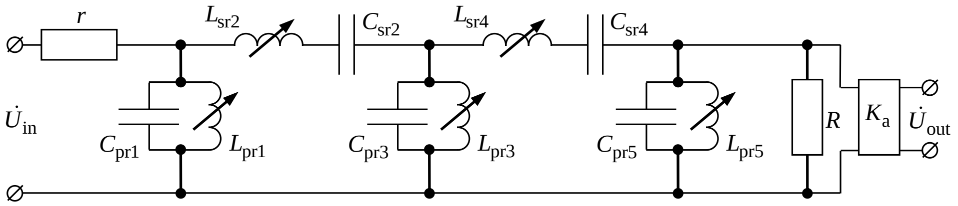

Fig. 6. Schematic diagram of the 10th-order BPF with variable inductors

frequency response $\widehat{H}^{(10)}_{\mathrm{BP_{pr}}C}(\omega_\mathrm{N})$ with a set of capacitors of standard values when tuning the filter centre frequency. The schematic diagram of the 10th-order BPF with variable inductors is shown in Fig. 6.

Assuming that $C'_{\mathrm{pr1}} = 3\ \mu\mathrm{F}$, then, with a centre frequency of 10^5 rad/s, the following inductance values are obtained:

$$L'_{\mathrm{pr1}} = 33.3\ \mu\mathrm{H},\ \ L'_{\mathrm{sr2}} = 9.1\ \mathrm{mH},$$
$$L'_{\mathrm{pr3}} = 25.6\ \mu\mathrm{H},\ \ L'_{\mathrm{sr4}} = 7.7\ \mathrm{mH},$$
$$L'_{\mathrm{pr5}} = 62.5\ \mu\mathrm{H}.$$

When tuning BPF in the frequency range $\omega_0 = (0.8\mathrm{K}\ 1.2)10^5$ rad/s $(-0.2 \le \Delta \le 0.2)$, the inductance overlap ratios are defined as:

$$k_L = (1+0.2)^2 \big/ (1-0.2)^2 = 2.25.$$

The amplifier gain and frequency response unevenness at the centre frequency over the entire tuning range are constant and equal to the initial values: $K_\mathrm{a} = 2.981,\ \widehat{\delta}_1 = 0.059.$

The function of normalised frequency $\omega \big/ \left[(1+\Delta)\omega_0\right] = \omega_\mathrm{N} / (1+\Delta)$ is defined as $\widehat{H}'^{(10)}_{\mathrm{BP}}\left[\omega_\mathrm{N}/(1+\Delta),\ (1+\Delta)\omega_0\right]$. The frequency responses $\widehat{H}'^{(10)}_{\mathrm{BP_{pr}}C}\left[\omega_\mathrm{N}/(1+\Delta),\ (1+\Delta)\omega_0\right]$ for various Δ are shown in Fig. 7. The solutions of the equations systems (10) and (11) for two extreme values of the frequency range corresponding to the retuning coefficient $\chi = (1+0.2)/(1-0.2) = 1.5$, are given in Table 3 (right columns).

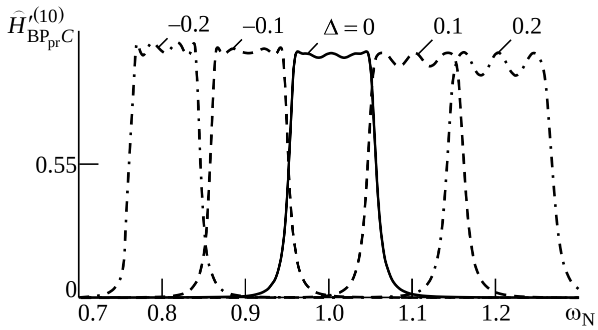

Fig. 7. Frequency responses of the 10th-order BPF with variable inductors for various Δ

It is possible to investigate the frequency response distortions when varying the BPF centre frequency with the use of variable capacitors. The inductances of parallel and sequential circuits are determined from correlations: $L_{\mathrm{pr}i} = 1\big/\left(\omega_0^2 C_{\mathrm{pr}i}\right)$, $L_{\mathrm{sr}k} = 1\big/\left(\omega_0^2 C_{\mathrm{sr}k}\right)$, where $C_{\mathrm{pr}i}$ and $C_{\mathrm{sr}k}$ are calculated values of the initial system of 6 equations:

$$L_{\mathrm{pr1}} = 33.43\ \mu\mathrm{H},\ \ L_{\mathrm{sr2}} = 9.31\ \mathrm{mH},$$
$$L_{\mathrm{pr3}} = 26.45\ \mu\mathrm{H},\ \ L_{\mathrm{sr4}} = 7.95\ \mathrm{mH},$$
$$L_{\mathrm{pr5}} = 61.25\ \mu\mathrm{H}.$$

A schematic diagram of the 10th-order BPF with variable capacitors is shown in Fig. 8, while the frequency responses $\widehat{H}'^{(10)}_{\mathrm{BP_{pr}}L}\left[\omega_\mathrm{N}/(1+\Delta),\ (1+\Delta)\omega_0\right]$ for the same Δ values are shown in Fig. 9. The curve for $\Delta = 0$ is an equiripple frequency response with parameters $K_\mathrm{a} = 2.978,\ \widehat{\delta}_1 = 0.059,$ which remain unchanged while tuning.

Solutions of the equations systems (10), (11) for two values of Δ are shown below. When $\Delta_1 = -0.2$,

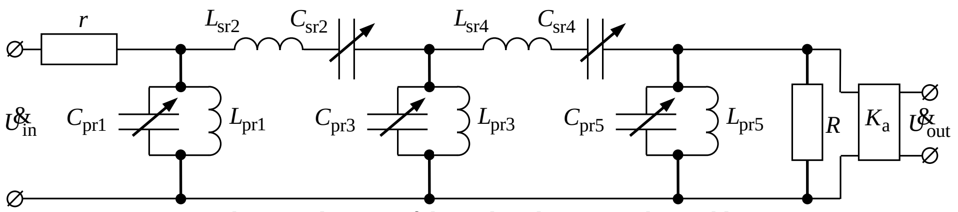

Fig. 8. Schematic diagram of the 10th-order BPF with variable capacitors

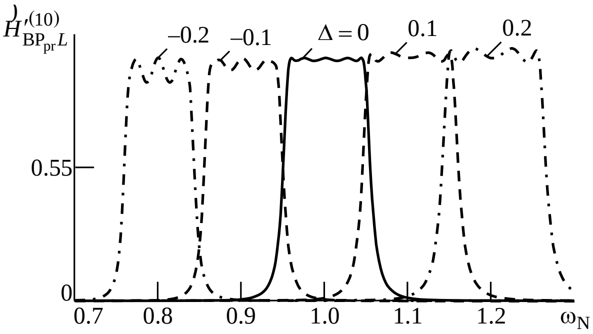

Fig. 9. Frequency responses of the 10th-order BPF with variable capacitors for various Δ

$q = 3$: $\hat{\delta}_2 = -0.869$, $\hat{\delta}_3 = 0.007$, $\hat{Q} = 10.006$, $\hat{\omega}_{\mathrm{N}2} = 0.345$, $\hat{\omega}_{\mathrm{N}3} = 0.674$. When $\Delta_\mathrm{u} = 0.2$, $q = 4$: $\hat{\delta}_2 = 0.389$, $\hat{\delta}_3 = -0.106$, $\hat{\delta}_4 = 0.317$, $\hat{Q} = 10.046$, $\hat{\omega}_{\mathrm{N}2} = 0.374$, $\hat{\omega}_{\mathrm{N}3} = 0.684$, $\hat{\omega}_{\mathrm{N}4} = 0.857$. The overlap ratio of capacitances is $k_C = 2.25$.

Fig. 10 shows the frequency response distortions of the BPF with variable inductors (see Fig. 6):

$$I^{(10)}(\Delta) = \int\limits_{\sqrt{1-1/(4Q^2)}-1/(2Q)}^{\sqrt{1-1/(4Q^2)}+1/(2Q)} \left\{ \tilde{H}^{(10)}_{\mathrm{BP}}(\omega_\mathrm{N}) - \hat{H}^{(10)}_{\mathrm{BP_{pr}}C}\left[\omega_\mathrm{N},\ (1+\Delta)\omega_0\right] \right\}^2 d\omega_\mathrm{N}$$

and with variable capacitors (Fig. 8):

$$J^{(10)}(\Delta) = \int\limits_{\sqrt{1-1/(4Q^2)}-1/(2Q)}^{\sqrt{1-1/(4Q^2)}+1/(2Q)} \left\{ \tilde{H}^{(10)}_{\mathrm{BP}}(\omega_\mathrm{N}) - \hat{H}^{(10)}_{\mathrm{BP_{pr}}L}\left[\omega_\mathrm{N},\ (1+\Delta)\omega_0\right] \right\}^2 d\omega_\mathrm{N},$$

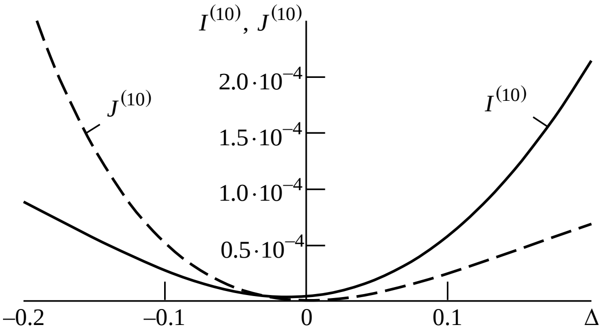

Fig. 10. Frequency response distortion functions of the tunable BPF

where

$$\tilde{H}^{(10)}_{\mathrm{BP}}(\omega_\mathrm{N}) = \left(\tilde{K}/Q^5\right)\omega_\mathrm{N}^5 \Big/ \sqrt{\tilde{\mathrm{M}}^{(10)}};$$

$\hat{H}^{(10)}_{\mathrm{BP_{pr}}C}\left[\omega_\mathrm{N},\ (1+\Delta)\omega_0\right]$ is a function of the normalised frequency ω_N, which is expressed by $C_{\mathrm{pr}i}$ and $C_{\mathrm{sr}k}$ (see Table 2), and the centre frequency $(1+\Delta)\omega_0$;

$$\hat{H}^{(10)}_{\mathrm{BP_{pr}}L}\left[\omega_\mathrm{N},\ (1+\Delta)\omega_0\right] = \frac{K_\mathrm{a}(1+\Delta)\omega_0 L_{\mathrm{pr}1}L_{\mathrm{pr}3}L_{\mathrm{pr}5}}{rL_{\mathrm{sr}2}L_{\mathrm{sr}4}}\omega_\mathrm{N}^5 \Big/ \sqrt{\hat{\mathrm{M}}^{(10)}_{\mathrm{BP_{pr}}L}},$$

where

$$\tilde{\mathrm{M}}^{(10)} = \left[\omega_\mathrm{N}^{10} + \left(5 + \frac{\tilde{b}_3}{Q^2}\right)\left(\omega_\mathrm{N}^8 - \omega_\mathrm{N}^2\right) + \left(10 + \frac{3\tilde{b}_3}{Q^2} + \frac{\tilde{b}_1}{Q^4}\right)\left(\omega_\mathrm{N}^6 - \omega_\mathrm{N}^4\right) - 1\right]^2 + \left[\frac{\tilde{b}_4}{Q}\left(\omega_\mathrm{N}^9 + \omega_\mathrm{N}\right) - \left(\frac{4\tilde{b}_4}{Q} + \frac{\tilde{b}_2}{Q^3}\right)\left(\omega_\mathrm{N}^7 + \omega_\mathrm{N}^3\right) + \left(\frac{6\tilde{b}_4}{Q} + \frac{2\tilde{b}_2}{Q^3} + \frac{\tilde{b}_0}{Q^5}\right)\omega_\mathrm{N}^5\right]^2;$$

$$\hat{\mathrm{M}}^{(10)}_{\mathrm{BP_{pr}}L} = \left(\omega_\mathrm{N}^{10} - \left\{5 + \frac{L_{\mathrm{pr}1} + L_{\mathrm{pr}3}}{L_{\mathrm{sr}2}} + \frac{L_{\mathrm{pr}3} + L_{\mathrm{pr}5}}{L_{\mathrm{sr}4}} + \frac{\left[(1+\Delta)\omega_0\right]^2 L_{\mathrm{pr}1}L_{\mathrm{pr}5}}{rR}\right\}\left(\omega_\mathrm{N}^8 - \omega_\mathrm{N}^2\right) + \left\{10 + 3\left(\frac{L_{\mathrm{pr}1} + L_{\mathrm{pr}3}}{L_{\mathrm{sr}2}} + \frac{L_{\mathrm{pr}3} + L_{\mathrm{pr}5}}{L_{\mathrm{sr}4}}\right) + \frac{L_{\mathrm{pr}1}\left(L_{\mathrm{pr}3} + L_{\mathrm{pr}5}\right) + L_{\mathrm{pr}3}L_{\mathrm{pr}5}}{L_{\mathrm{sr}2}L_{\mathrm{sr}4}} + \frac{\left[(1+\Delta)\omega_0\right]^2 L_{\mathrm{pr}1}L_{\mathrm{pr}3}L_{\mathrm{pr}5}}{rR} \times \left(\frac{1}{L_{\mathrm{sr}2}} + \frac{3}{L_{\mathrm{sr}3}} + \frac{1}{L_{\mathrm{sr}4}}\right)\right\}\left(\omega_\mathrm{N}^6 - \omega_\mathrm{N}^4\right) - 1\right)^2 + \left[(1+\Delta)\omega_0\right]^2 \left\{\frac{rL_{\mathrm{pr}5} + RL_{\mathrm{pr}1}}{rR}\left(\omega_\mathrm{N}^9 + \omega_\mathrm{N}\right) - \right.$$

$$-\left[\left(\frac{L_{\text{pr1}}}{r}+\frac{L_{\text{pr5}}}{R}\right)\left(4+\frac{L_{\text{pr3}}}{L_{\text{sr2}}}+\frac{L_{\text{pr3}}}{L_{\text{sr4}}}\right)+\right.$$
$$\left.+L_{\text{pr1}}\frac{rL_{\text{sr4}}+RL_{\text{sr2}}}{rRL_{\text{sr2}}L_{\text{sr4}}}L_{\text{pr5}}\right]\left(\omega_{\text{N}}^{7}+\omega_{\text{N}}^{3}\right)+$$
$$+\left[2\frac{rL_{\text{pr5}}+RL_{\text{pr1}}}{rR}\left(3+\frac{L_{\text{pr3}}}{L_{\text{sr2}}}+\frac{L_{\text{pr3}}}{L_{\text{sr4}}}\right)+\right.$$
$$+2L_{\text{pr1}}\left(\frac{1}{RL_{\text{sr2}}}+\frac{1}{rL_{\text{sr4}}}\right)L_{\text{pr5}}+$$
$$\left.\left.+\frac{(r+R)L_{\text{pr1}}L_{\text{pr3}}L_{\text{pr5}}}{rRL_{\text{sr2}}L_{\text{sr4}}}\right]\omega_{\text{N}}^{5}\right\}^{2}.$$

The average values of functions $I^{(10)}(\Delta)$ and $J^{(10)}(\Delta)$ over the argument range $[-0.2\le\Delta\le 0.2]$ are equal to $54.92\cdot 10^{-6}$ and $53.51\cdot 10^{-6}$, respectively. From the function $I^{(10)}(\Delta)$, it can be seen that the frequency response of a non-tunable BPF can be corrected by a slight change in frequency ω_0 using inductors. The function minima occur at $\Delta=-0.0144$, which corresponds to the corrected angular frequency of circuits tuning $\omega_{0\text{cor}}=(1-0.0144)\omega_0=98\,560$ rad/s. The comparison of the initial $\widehat{H}^{(10)}_{\text{BP}_{\text{pr}}C}[\omega_{\text{N}},\ \omega_0]$ and corrected $\widehat{H}'^{(10)}_{\text{BP}_{\text{pr}}C}[\omega_{\text{N}}/0.9856,\ \omega_{0\text{cor}}]$ frequency responses (Fig. 11) shows that the corrected frequency response has smaller ripple values at minima points, while at maxima points the ripple values are unchanged.

Band-rejection filters. When converting from LPF to BRF with a centre rejection frequency of ω_0, capacitance C is replaced by a series resonant circuit with elements $C_{\text{sr}}=C/\Theta$ and $L_{\text{sr}}=\Theta/(\omega_0^2 C)$, while inductance L is replaced by a parallel resonant circuit with elements $L_{\text{pr}}=L/\Theta$ and $C_{\text{pr}}=\Theta/(\omega_0^2 L)$. When $H^{(n)}(1)=1/\sqrt{2}$, then Θ is a BRF quality factor that is equal to the ratio of the normalised centre rejection frequency to the normalised stopband, which is determined at the level of $1/\sqrt{2}$ the TF absolute value.

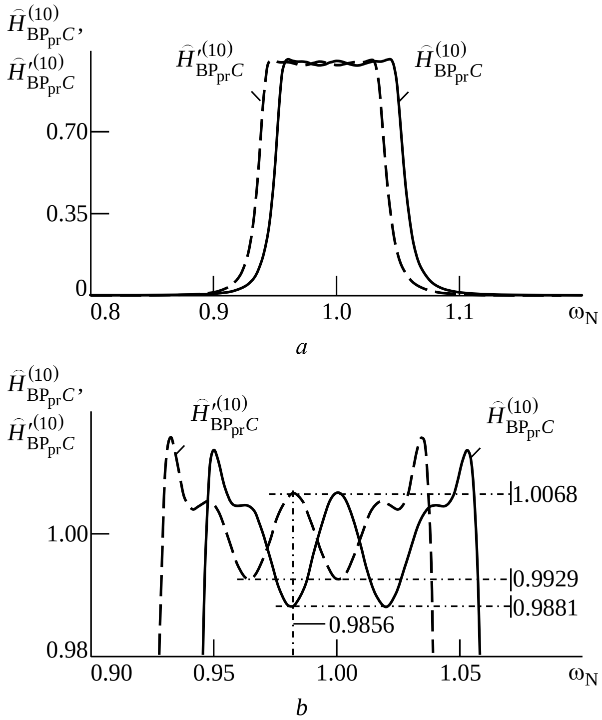

Fig. 11. Initial and corrected frequency responses of BPF (*a*); central parts of BPF frequency responses (*b*)

Fig. 12 and 13 show schematic diagrams of the $2n^{\text{th}}$-order BRF with the series resonant circuit in the transverse branch at the input (Fig. 12) and the parallel resonant circuit in the longitudinal branch at the input (Fig. 13) with elements $C_{\text{sr}\,i}$, $L_{\text{sr}\,i}$, $C_{\text{pr}\,k}$, $L_{\text{pr}\,k}$, $i,\ k=\overline{1,\ n}$.

Table 4 shows the TF of LPF prototypes of BRF $H^{(n)}_{\text{r}\,C}(s_{\text{N}})$ (schematic diagram in Fig. 1) and $H^{(n)}_{\text{r}\,L}(s_{\text{N}})$ (schematic diagram in Fig. 2) with filter orders $n=\overline{1,\ 5}$, which are expressed through the circuits capacitance $C_{\text{sr}\,i}$, $C_{\text{pr}\,k}$.

Table 5 shows the $2n^{\text{th}}$-order BRF frequency responses of $H^{(2n)}_{\text{BR}_{\text{sr}}C}(\omega_{\text{N}})$ and $H^{(2n)}_{\text{BR}_{\text{pr}}C}(\omega_{\text{N}})$ for $2n=2,\ 4,\ 6,\ 8,\ 10$ with the series and parallel reso-

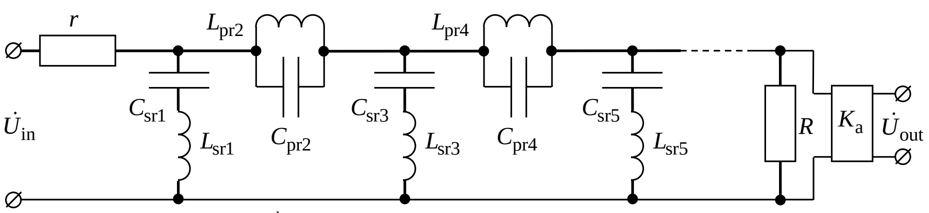

Fig. 12. Schematic diagram of the $2n^{\text{th}}$-order BRF with the series resonant circuit in the transverse branch at the input

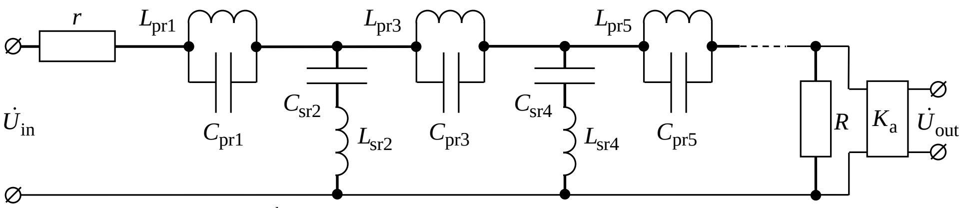

Fig. 13. Schematic diagram of the $2n^{\text{th}}$-order BRF with the parallel resonant circuit in the longitudinal branch at the input

nant circuits at the input and corresponding to Fig. 12 and 13, respectively. Frequency responses are expressed through the circuit capacitances $C_{\text{sr}\,i}$, $C_{\text{pr}\,k}$.

Fig. 14, *a* shows the equiripple frequency responses of the 5th-order LPF prototype $\tilde{H}^{(5)}_{\text{r}\,C}(\omega_{\text{N}})$ (see Fig. 1) and the 10th-order BRF with the series resonant circuit in the transverse branch at the input $\tilde{H}^{(10)}_{\text{BR}_{\text{sr}}C}(\omega_{\text{N}})$ (Fig. 12) with circuits frequencies tuned at $\omega_0 = 10^5$ rad/s and frequency response parameters: $\widehat{\delta} = 0.1$, $Q = 10$. The magnified central parts of the frequency responses are shown in Fig. 14, *b*. When $r = 100\ \Omega$, $R = 51\ \Omega$, then BRF elements have the following non-standard values:

$$C_{\text{sr1}} = 29.92\ \text{nF},\ \ L_{\text{sr1}} = 3.34\ \text{mH},$$

$$C_{\text{pr2}} = 1.07\ \mu\text{F},\ \ L_{\text{pr2}} = 93.12\ \mu\text{H},$$

$$C_{\text{sr3}} = 37.81\ \text{nF},\ \ L_{\text{sr3}} = 2.65\ \text{mH},$$

$$C_{\text{pr4}} = 1.26\ \mu\text{F},\ \ L_{\text{pr4}} = 79.53\ \mu\text{H},$$

$$C_{\text{sr5}} = 16.33\ \text{nF},\ \ L_{\text{sr5}} = 6.12\ \text{mH}.$$

The amplifier gain is $K_{\text{a}} = 2.978$.

Table 4. Transfer functions of LPF prototypes of BRF

$n = 1$
$H^{(1)}_{\text{r}\,C}(s_{\text{N}}) = \dfrac{K_{\text{a}}}{\omega_0 \Theta r C_{\text{sr1}}} \Big/ \left(s_{\text{N}} + \dfrac{r+R}{\omega_0 \Theta r C_{\text{sr1}} R}\right)$; $H^{(1)}_{\text{r}\,L}(s_{\text{N}}) = \dfrac{K_{\text{a}}\, \omega_0 R C_{\text{pr1}}}{\Theta} \Big/ \left[s_{\text{N}} + \dfrac{\omega_0 (r+R) C_{\text{pr1}}}{\Theta}\right]$
$n = 2$
$H^{(2)}_{\text{r}\,C}(s_{\text{N}}) = \dfrac{K_{\text{a}} R C_{\text{pr2}}}{\Theta^2 r C_{\text{sr1}}} \Big/ \left[s_{\text{N}}^2 + \dfrac{1+\omega_0^2 r C_{\text{sr1}} R C_{\text{pr2}}}{\Theta \omega_0 r C_{\text{sr1}}} s_{\text{N}} + \dfrac{(r+R) C_{\text{pr2}}}{\Theta^2 r C_{\text{sr1}}}\right]$; $H^{(2)}_{\text{r}\,L}(s_{\text{N}}) = \dfrac{K_{\text{a}} C_{\text{pr1}}}{\Theta^2 C_{\text{sr2}}} \Big/ \left[s_{\text{N}}^2 + \dfrac{1+\omega_0^2 r C_{\text{pr1}} R C_{\text{sr2}}}{\Theta \omega_0 R C_{\text{sr2}}} s_{\text{N}} + \dfrac{(r+R) C_{\text{pr1}}}{\Theta^2 R C_{\text{sr2}}}\right]$
$n = 3$
$H^{(3)}_{\text{r}\,C}(s_{\text{N}}) = \dfrac{K_{\text{a}} C_{\text{pr2}} \big/ \left(\Theta^3 \omega_0 r C_{\text{sr1}} C_{\text{sr3}}\right)}{s_{\text{N}}^3 + \dfrac{r C_{\text{sr1}} + R C_{\text{sr3}}}{\Theta \omega_0 r C_{\text{sr1}} R C_{\text{sr3}}} s_{\text{N}}^2 + \dfrac{1+\omega_0^2 r (C_{\text{sr1}} + C_{\text{sr3}}) R C_{\text{pr2}}}{\Theta^2 \omega_0^2\ r C_{\text{sr1}} R C_{\text{sr3}}} s_{\text{н}} + \dfrac{(r+R) C_{\text{pr2}}}{\Theta^3 \omega_0 r C_{\text{sr1}} R C_{\text{sr3}}}}$; $H^{(3)}_{\text{r}\,L}(s_{\text{N}}) = \dfrac{K_{\text{a}} \omega_0 R C_{\text{pr1}} C_{\text{pr3}} \big/ \left(\Theta^3 C_{\text{sr2}}\right)}{s_{\text{N}}^3 + \dfrac{\omega_0 \left(r C_{\text{pr1}} + R C_{\text{pr3}}\right)}{\Theta} s_{\text{N}}^2 + \dfrac{C_{\text{pr1}} + C_{\text{pr3}} \left(1+\omega_0^2 r C_{\text{pr1}} R C_{\text{sr2}}\right)}{\Theta^2 C_{\text{sr2}}} s_{\text{N}} + \dfrac{\omega_0 (r+R) C_{\text{pr1}} C_{\text{pr3}}}{\Theta^3 C_{\text{sr2}}}}$
$n = 4$
$H^{(4)}_{\text{r}\,C}(s_{\text{N}}) = \left[K_{\text{a}} R C_{\text{pr2}} C_{\text{pr4}} \big/ \left(\Theta^4 r C_{\text{sr1}} C_{\text{sr3}}\right)\right] \big/ \Lambda^{(4)}_{\text{r}\,C}$, where $\Lambda^{(4)}_{\text{r}\,C} = s_{\text{N}}^4 + \dfrac{1+\omega_0^2 r C_{\text{sr1}} R C_{\text{pr4}}}{\Theta \omega_0 r C_{\text{sr1}}} s_{\text{N}}^3 + \dfrac{r\left(C_{\text{sr1}} + C_{\text{sr3}}\right) C_{\text{pr2}} + \left(r C_{\text{sr1}} + R C_{\text{sr3}}\right) C_{\text{pr4}}}{\Theta^2 r C_{\text{sr1}} C_{\text{sr3}}} s_{\text{N}}^2 + \dfrac{C_{\text{pr2}} + C_{\text{pr4}} + \omega_0^2 r \left(C_{\text{sr1}} + C_{\text{sr3}}\right) R C_{\text{pr2}} C_{\text{pr4}}}{\Theta^3 \omega_0 r C_{\text{sr1}} C_{\text{sr3}}} s_{\text{N}} + \dfrac{(r+R) C_{\text{pr2}} C_{\text{pr4}}}{\Theta^4 r C_{\text{sr1}} C_{\text{sr3}}}$;

End of Table 4

$$H_{\mathrm{r}\,L}^{(4)}(s_{\mathrm{N}})=\left[K_{\mathrm{a}}C_{\mathrm{pr1}}C_{\mathrm{pr3}}\big/\left(\Theta^{4}C_{\mathrm{sr2}}C_{\mathrm{sr4}}\right)\right]\big/\Lambda_{\mathrm{r}\,L}^{(4)},$$ where $$\Lambda_{\mathrm{r}\,L}^{(4)}=s_{\mathrm{N}}^{4}+\frac{1+\omega_0^2 rC_{\mathrm{pr1}}RC_{\mathrm{sr4}}}{\Theta\omega_0 RC_{\mathrm{sr4}}}s_{\mathrm{N}}^{3}+\frac{\left(rC_{\mathrm{pr1}}+RC_{\mathrm{pr3}}\right)C_{\mathrm{sr2}}+R\left(C_{\mathrm{pr1}}+C_{\mathrm{pr3}}\right)C_{\mathrm{sr4}}}{\Theta^{2}RC_{\mathrm{sr2}}C_{\mathrm{sr4}}}s_{\mathrm{N}}^{2}+ +\frac{C_{\mathrm{pr1}}+C_{\mathrm{pr3}}+\omega_0^2 rC_{\mathrm{pr1}}R\left(C_{\mathrm{sr2}}+C_{\mathrm{sr4}}\right)C_{\mathrm{pr3}}}{\Theta^{3}\omega_0 RC_{\mathrm{sr2}}C_{\mathrm{sr4}}}s_{\mathrm{N}}+\frac{(r+R)C_{\mathrm{pr1}}C_{\mathrm{pr3}}}{\Theta^{4}RC_{\mathrm{sr2}}C_{\mathrm{sr4}}};$$
$n=5$
$$H_{\mathrm{r}\,C}^{(5)}(s_{\mathrm{N}})=\left[K_{\mathrm{a}}C_{\mathrm{pr2}}C_{\mathrm{pr4}}\big/\left(\Theta^{5}\omega_0 rC_{\mathrm{sr1}}C_{\mathrm{sr3}}C_{\mathrm{sr5}}\right)\right]\big/\Lambda_{\mathrm{r}\,C}^{(5)},$$ where $$\Lambda_{\mathrm{r}\,C}^{(5)}=s_{\mathrm{N}}^{5}+\frac{rC_{\mathrm{sr1}}+RC_{\mathrm{sr5}}}{\Theta\omega_0 rC_{\mathrm{sr1}}RC_{\mathrm{sr5}}}s_{\mathrm{N}}^{4}+\left(\frac{C_{\mathrm{sr1}}C_{\mathrm{pr4}}+C_{\mathrm{pr2}}C_{\mathrm{sr5}}}{\Theta^{2}C_{\mathrm{sr1}}C_{\mathrm{sr5}}}+\frac{C_{\mathrm{pr2}}+C_{\mathrm{pr4}}}{\Theta^{2}C_{\mathrm{sr3}}}+\frac{1}{\Theta^{2}\omega_0^2 rC_{\mathrm{sr1}}RC_{\mathrm{sr5}}}\right)s_{\mathrm{N}}^{3}+ +\frac{\left(rC_{\mathrm{sr1}}+RC_{\mathrm{sr5}}\right)\left(C_{\mathrm{pr2}}+C_{\mathrm{pr4}}\right)+\left(rC_{\mathrm{pr2}}+RC_{\mathrm{pr4}}\right)C_{\mathrm{sr3}}}{\Theta^{3}\omega_0 rC_{\mathrm{sr1}}RC_{\mathrm{sr3}}C_{\mathrm{sr5}}}s_{\mathrm{N}}^{2}+\frac{C_{\mathrm{pr2}}+C_{\mathrm{pr4}}+\omega_0^2 r\left(C_{\mathrm{sr1}}+C_{\mathrm{sr3}}+C_{\mathrm{sr5}}\right)RC_{\mathrm{pr2}}C_{\mathrm{pr4}}}{\Theta^{4}\omega_0^2 rC_{\mathrm{sr1}}RC_{\mathrm{sr3}}C_{\mathrm{sr5}}}s_{\mathrm{N}}+ +\frac{(r+R)C_{\mathrm{pr2}}C_{\mathrm{pr4}}}{\Theta^{5}\omega_0 rC_{\mathrm{sr1}}RC_{\mathrm{sr3}}C_{\mathrm{sr5}}};$$ $$H_{\mathrm{r}\,L}^{(5)}(s_{\mathrm{N}})=\left[K_{\mathrm{a}}\omega_0 RC_{\mathrm{pr1}}C_{\mathrm{pr3}}C_{\mathrm{pr5}}\big/\left(\Theta^{5}C_{\mathrm{sr2}}C_{\mathrm{sr4}}\right)\right]\big/\Lambda_{\mathrm{r}\,L}^{(5)},$$ where $$\Lambda_{\mathrm{r}\,L}^{(5)}=s_{\mathrm{N}}^{5}+\frac{\omega_0\left(rC_{\mathrm{pr1}}+RC_{\mathrm{pr5}}\right)}{\Theta}s_{\mathrm{N}}^{4}+\left(\frac{C_{\mathrm{pr1}}+C_{\mathrm{pr3}}}{\Theta^{2}C_{\mathrm{sr2}}}+\frac{C_{\mathrm{pr3}}+C_{\mathrm{pr5}}}{\Theta^{2}C_{\mathrm{sr4}}}+\frac{\omega_0^2 rC_{\mathrm{pr1}}RC_{\mathrm{pr5}}}{\Theta^{2}}\right)s_{\mathrm{N}}^{3}+ +\frac{\omega_0}{\Theta^{3}}\left[C_{\mathrm{pr1}}\left(\frac{r}{C_{\mathrm{sr4}}}+\frac{R}{C_{\mathrm{sr2}}}\right)C_{\mathrm{pr5}}+\frac{\left(C_{\mathrm{sr2}}+C_{\mathrm{sr4}}\right)C_{\mathrm{pr3}}}{C_{\mathrm{sr2}}C_{\mathrm{sr4}}}\left(rC_{\mathrm{pr1}}+RC_{\mathrm{pr5}}\right)\right]s_{\mathrm{N}}^{2}+ +\frac{C_{\mathrm{pr1}}\left(C_{\mathrm{pr3}}+C_{\mathrm{pr5}}\right)+C_{\mathrm{pr3}}C_{\mathrm{pr5}}\left[1+\omega_0^2 rC_{\mathrm{pr1}}R\left(C_{\mathrm{sr2}}+C_{\mathrm{sr4}}\right)\right]}{\Theta^{4}C_{\mathrm{sr2}}C_{\mathrm{sr4}}}s_{\mathrm{N}}+\frac{\omega_0(r+R)C_{\mathrm{pr1}}C_{\mathrm{pr3}}C_{\mathrm{pr5}}}{\Theta^{5}C_{\mathrm{sr2}}C_{\mathrm{sr4}}}$$

Table 5. BRF frequency responses

$2n=2$
$$H_{\mathrm{BR_{sr}}C}^{(2)}(\omega_{\mathrm{N}})=\frac{K_{\mathrm{a}}R}{r+R}\left\|\omega_{\mathrm{N}}^{2}-1\right\|\Big/\sqrt{\left(\omega_{\mathrm{N}}^{2}-1\right)^{2}+\left(\frac{\omega_0 rC_{\mathrm{sr1}}R}{r+R}\omega_{\mathrm{N}}\right)^{2}};\quad H_{\mathrm{BR_{pr}}C}^{(2)}(\omega_{\mathrm{N}})=\frac{K_{\mathrm{a}}R}{r+R}\left\|\omega_{\mathrm{N}}^{2}-1\right\|\Big/\sqrt{\left(\omega_{\mathrm{N}}^{2}-1\right)^{2}+\left[\frac{1}{\omega_0(r+R)C_{\mathrm{pr1}}}\omega_{\mathrm{N}}\right]^{2}}$$
$2n=4$
$$H_{\mathrm{BR_{sr}}C}^{(4)}(\omega_{\mathrm{N}})=\frac{K_{\mathrm{a}}R}{r+R}\left(\omega_{\mathrm{N}}^{2}-1\right)^{2}\Big/\sqrt{\left\{\omega_{\mathrm{N}}^{4}-\left[2+\frac{rC_{\mathrm{sr1}}}{(r+R)C_{\mathrm{pr2}}}\right]\omega_{\mathrm{N}}^{2}+1\right\}^{2}+\left[\frac{\omega_0 rR}{r+R}\left(C_{\mathrm{sr1}}+\frac{1}{\omega_0^2 rC_{\mathrm{pr2}}R}\right)\left(\omega_{\mathrm{N}}^{3}-\omega_{\mathrm{N}}\right)\right]^{2}};$$ $$H_{\mathrm{BR_{pr}}C}^{(4)}(\omega_{\mathrm{N}})=\frac{K_{\mathrm{a}}R}{r+R}\left(\omega_{\mathrm{N}}^{2}-1\right)^{2}\Big/\sqrt{\left\{\omega_{\mathrm{N}}^{4}-\left[2+\frac{RC_{\mathrm{sr2}}}{(r+R)C_{\mathrm{pr1}}}\right]\omega_{\mathrm{N}}^{2}+1\right\}^{2}+\left[\frac{\omega_0 rR}{r+R}\left(C_{\mathrm{sr2}}+\frac{1}{\omega_0^2 rC_{\mathrm{pr1}}R}\right)\left(\omega_{\mathrm{N}}^{3}-\omega_{\mathrm{N}}\right)\right]^{2}}$$
$2n=6$
$$H_{\mathrm{BR_{sr}}C}^{(6)}(\omega_{\mathrm{N}})=\frac{K_{\mathrm{a}}R}{r+R}\left\|\left(\omega_{\mathrm{N}}^{2}-1\right)^{3}\right\|\Big/\sqrt{\mathrm{M}_{\mathrm{BR_{sr}}C}^{(6)}},$$ where $$\mathrm{M}_{\mathrm{BR_{sr}}C}^{(6)}=\left\{\omega_{\mathrm{N}}^{6}-\left[3+\frac{rC_{\mathrm{sr1}}+RC_{\mathrm{sr3}}}{(r+R)C_{\mathrm{pr2}}}\right]\left(\omega_{\mathrm{N}}^{4}-\omega_{\mathrm{N}}^{2}\right)-1\right\}^{2}+ +\left(\frac{\omega_0 rR}{r+R}\right)^{2}\left[\left(C_{\mathrm{sr1}}+C_{\mathrm{sr3}}+\frac{1}{\omega_0^2 rRC_{\mathrm{pr2}}}\right)\left(\omega_{\mathrm{N}}^{5}+\omega_{\mathrm{N}}\right)-2\left(C_{\mathrm{sr1}}+C_{\mathrm{sr3}}+\frac{C_{\mathrm{sr1}}C_{\mathrm{sr3}}}{2C_{\mathrm{pr2}}}+\frac{1}{\omega_0^2 rRC_{\mathrm{pr2}}}\right)\omega_{\mathrm{N}}^{3}\right]^{2};$$

Continued of the Table 5

$$H^{(6)}_{\mathrm{BR_{pr}}C}(\omega_\mathrm{N})=\frac{K_\mathrm{a}R}{r+R}\left|\left(\omega_\mathrm{N}^2-1\right)^3\right|\Big/\sqrt{\mathrm{M}^{(6)}_{\mathrm{BR_{pr}}C}},$$

where

$$\mathrm{M}^{(6)}_{\mathrm{BR_{pr}}C}=\left\{\omega_\mathrm{N}^6-\left[3+\frac{\left(rC_\mathrm{pr1}+RC_\mathrm{pr3}\right)C_\mathrm{sr2}}{(r+R)C_\mathrm{pr1}C_\mathrm{pr3}}\right]\left(\omega_\mathrm{N}^4-\omega_\mathrm{N}^2\right)-1\right\}^2+$$
$$+\left(\frac{\omega_0 rR}{r+R}\right)^2\left\{\left(C_\mathrm{sr2}+\frac{C_\mathrm{pr1}+C_\mathrm{pr3}}{\omega_0^2 rC_\mathrm{pr1}RC_\mathrm{pr3}}\right)\left(\omega_\mathrm{N}^5+\omega_\mathrm{N}\right)-2\left[C_\mathrm{sr2}+\frac{2\left(C_\mathrm{pr1}+C_\mathrm{pr3}\right)+C_\mathrm{sr2}}{2\omega_0^2 rC_\mathrm{pr1}RC_\mathrm{pr3}}\right]\omega_\mathrm{N}^3\right\}^2$$

$2n=8$

$$H^{(8)}_{\mathrm{BR_{sr}}C}(\omega_\mathrm{N})=\left[K_\mathrm{a}R/(r+R)\right]\left(\omega_\mathrm{N}^2-1\right)^4\Big/\sqrt{\mathrm{M}^{(8)}_{\mathrm{BR_{sr}}C}},$$

where

$$\mathrm{M}^{(8)}_{\mathrm{BR_{sr}}C}=\left\{\omega_\mathrm{N}^8-\left[4+\frac{rC_\mathrm{sr1}+RC_\mathrm{sr3}}{(r+R)C_\mathrm{pr2}}+\frac{r\left(C_\mathrm{sr1}+C_\mathrm{sr3}\right)}{(r+R)C_\mathrm{pr4}}\right]\left(\omega_\mathrm{N}^6+\omega_\mathrm{N}^2\right)+\right.$$
$$\left.+\left[6+\frac{rC_\mathrm{sr1}\left(2C_\mathrm{pr2}+C_\mathrm{sr3}+2C_\mathrm{pr4}\right)+2\left(rC_\mathrm{pr2}+RC_\mathrm{pr4}\right)C_\mathrm{sr3}}{(r+R)C_\mathrm{pr2}C_\mathrm{pr4}}\right]\omega_\mathrm{N}^4+1\right\}^2+\left(\frac{\omega_0 rR}{r+R}\right)^2\times$$
$$\times\left\{\left(C_\mathrm{sr1}+C_\mathrm{sr3}+\frac{C_\mathrm{pr2}+C_\mathrm{pr4}}{\omega_0^2 rC_\mathrm{pr2}RC_\mathrm{pr4}}\right)\left(\omega_\mathrm{N}^7-\omega_\mathrm{N}\right)-3\left[C_\mathrm{sr1}+C_\mathrm{sr3}+\frac{C_\mathrm{sr1}C_\mathrm{sr3}}{3C_\mathrm{pr2}}+\frac{3\left(C_\mathrm{pr2}+C_\mathrm{pr4}\right)+C_\mathrm{sr3}}{3\omega_0^2 rC_\mathrm{pr2}RC_\mathrm{pr4}}\right]\left(\omega_\mathrm{N}^5-\omega_\mathrm{N}^3\right)\right\}^2;$$

$$H^{(8)}_{\mathrm{BR_{pr}}C}(\omega_\mathrm{N})=\left[K_\mathrm{a}R/(r+R)\right]\left(\omega_\mathrm{N}^2-1\right)^4\Big/\sqrt{\mathrm{M}^{(8)}_{\mathrm{BR_{pr}}C}},$$

where

$$\mathrm{M}^{(8)}_{\mathrm{BR_{pr}}C}=\left\{\omega_\mathrm{N}^8-\left[4+\frac{rC_\mathrm{sr2}+RC_\mathrm{sr4}}{(r+R)C_\mathrm{pr3}}+\frac{R\left(C_\mathrm{sr2}+C_\mathrm{sr4}\right)}{(r+R)C_\mathrm{pr1}}\right]\left(\omega_\mathrm{N}^6+\omega_\mathrm{N}^2\right)+\right.$$
$$\left.+\left[6+\frac{2\left(rC_\mathrm{pr1}+RC_\mathrm{pr3}\right)C_\mathrm{sr2}+R\left(2C_\mathrm{pr1}+C_\mathrm{sr2}+2C_\mathrm{pr3}\right)C_\mathrm{sr4}}{(r+R)C_\mathrm{pr1}C_\mathrm{pr3}}\right]\omega_\mathrm{N}^4+1\right\}^2+\left(\frac{\omega_0 rR}{r+R}\right)^2\times$$
$$\times\left\{\left(C_\mathrm{sr2}+C_\mathrm{sr4}+\frac{C_\mathrm{pr1}+C_\mathrm{pr3}}{\omega_0^2 rC_\mathrm{pr1}RC_\mathrm{pr3}}\right)\left(\omega_\mathrm{N}^7-\omega_\mathrm{N}\right)-3\left[C_\mathrm{sr2}+C_\mathrm{sr4}+\frac{C_\mathrm{sr2}C_\mathrm{sr4}}{3C_\mathrm{pr3}}+\frac{3\left(C_\mathrm{pr1}+C_\mathrm{pr3}\right)+C_\mathrm{sr2}}{3\omega_0^2 rC_\mathrm{pr1}RC_\mathrm{pr3}}\right]\left(\omega_\mathrm{N}^5-\omega_\mathrm{N}^3\right)\right\}^2$$

$2n=10$

$$H^{(10)}_{\mathrm{BR_{sr}}C}(\omega_\mathrm{N})=\left[K_\mathrm{a}R/(r+R)\right]\left|\left(\omega_\mathrm{N}^2-1\right)^5\right|\Big/\sqrt{\mathrm{M}^{(10)}_{\mathrm{BR_{sr}}C}},$$

where

$$\mathrm{M}^{(10)}_{\mathrm{BR_{sr}}C}=\left\{\omega_\mathrm{N}^{10}-\left[5+\frac{rC_\mathrm{sr1}+R\left(C_\mathrm{sr3}+C_\mathrm{sr5}\right)}{(r+R)C_\mathrm{pr2}}+\frac{r\left(C_\mathrm{sr1}+C_\mathrm{sr3}\right)+RC_\mathrm{sr5}}{(r+R)C_\mathrm{pr4}}\right]\left(\omega_\mathrm{N}^8-\omega_\mathrm{N}^2\right)+\right.$$
$$\left.+\left[10+3\frac{rC_\mathrm{sr1}+R\left(C_\mathrm{sr3}+C_\mathrm{sr5}\right)}{(r+R)C_\mathrm{pr2}}+3\frac{r\left(C_\mathrm{sr1}+C_\mathrm{sr3}\right)+RC_\mathrm{sr5}}{(r+R)C_\mathrm{pr4}}+\frac{\left(rC_\mathrm{sr1}+RC_\mathrm{sr5}\right)C_\mathrm{sr3}}{(r+R)C_\mathrm{pr2}C_\mathrm{pr4}}\right]\left(\omega_\mathrm{N}^6-\omega_\mathrm{N}^4\right)-1\right\}^2+$$
$$+\left(\frac{\omega_0 rR}{r+R}\right)^2\left\{\left(C_\mathrm{sr1}+C_\mathrm{sr3}+C_\mathrm{sr5}+\frac{C_\mathrm{pr2}+C_\mathrm{pr4}}{\omega_0^2 rC_\mathrm{pr2}RC_\mathrm{pr4}}\right)\left(\omega_\mathrm{N}^9+\omega_\mathrm{N}\right)-\right.$$
$$-4\left[C_\mathrm{sr1}+C_\mathrm{sr3}+C_\mathrm{sr5}+\frac{C_\mathrm{sr1}\left(C_\mathrm{sr3}+C_\mathrm{sr5}\right)}{4C_\mathrm{pr2}}+\frac{\left(C_\mathrm{sr1}+C_\mathrm{sr3}\right)C_\mathrm{sr5}}{4C_\mathrm{pr4}}+\frac{4C_\mathrm{pr2}+C_\mathrm{sr3}+4C_\mathrm{pr4}}{4\omega_0^2 rC_\mathrm{pr2}RC_\mathrm{pr4}}\right]\left(\omega_\mathrm{N}^7+\omega_\mathrm{N}^3\right)+$$
$$\left.+6\left[C_\mathrm{sr1}+C_\mathrm{sr3}+C_\mathrm{sr5}+\frac{C_\mathrm{sr1}\left(C_\mathrm{sr3}+C_\mathrm{sr5}\right)}{3C_\mathrm{pr2}}+\frac{\left(C_\mathrm{sr1}+C_\mathrm{sr3}\right)C_\mathrm{sr5}}{3C_\mathrm{pr4}}+\frac{C_\mathrm{sr1}C_\mathrm{sr3}C_\mathrm{sr5}}{6C_\mathrm{pr2}C_\mathrm{pr4}}+\frac{3\left(C_\mathrm{pr2}+C_\mathrm{pr4}\right)+C_\mathrm{sr3}}{3\omega_0^2 rC_\mathrm{pr2}RC_\mathrm{pr4}}\right]\omega_\mathrm{N}^5\right\}^2;$$

End of Table 5

$$H^{(10)}_{\mathrm{BR}_{\mathrm{pr}}C}(\omega_{\mathrm{N}}) = \left[K_{\mathrm{a}} R/(r+R)\right]\left|\left(\omega_{\mathrm{N}}^2 - 1\right)^5\right| \Big/ \sqrt{\mathrm{M}^{(10)}_{\mathrm{BR}_{\mathrm{pr}}C}},$$

where

$$\mathrm{M}^{(10)}_{\mathrm{BR}_{\mathrm{pr}}C} = \left\{\omega_{\mathrm{N}}^{10} - \left[5 + \frac{rC_{\mathrm{sr}2} + RC_{\mathrm{sr}4}}{(r+R)C_{\mathrm{pr}3}} + \frac{\left(rC_{\mathrm{pr}1} + RC_{\mathrm{pr}5}\right)\left(C_{\mathrm{sr}2} + C_{\mathrm{sr}4}\right)}{(r+R)C_{\mathrm{pr}1}C_{\mathrm{pr}5}}\right]\left(\omega_{\mathrm{N}}^8 - \omega_{\mathrm{N}}^2\right) + \right.$$
$$\left. + \left[10 + 3\frac{rC_{\mathrm{sr}2} + RC_{\mathrm{sr}4}}{(r+R)C_{\mathrm{pr}3}} + 3\frac{\left(rC_{\mathrm{pr}1} + RC_{\mathrm{pr}5}\right)\left(C_{\mathrm{sr}2} + C_{\mathrm{sr}4}\right)}{(r+R)C_{\mathrm{pr}1}C_{\mathrm{pr}5}} + \frac{\left(rC_{\mathrm{pr}1} + RC_{\mathrm{pr}5}\right)C_{\mathrm{sr}2}C_{\mathrm{sr}4}}{(r+R)C_{\mathrm{pr}1}C_{\mathrm{pr}3}C_{\mathrm{pr}5}}\right]\left(\omega_{\mathrm{N}}^6 - \omega_{\mathrm{N}}^4\right) - 1\right\}^2 +$$
$$+ \left(\frac{\omega_0 rR}{r+R}\right)^2 \left\{\left[C_{\mathrm{sr}2} + C_{\mathrm{sr}4} + \frac{C_{\mathrm{pr}1}C_{\mathrm{pr}3} + \left(C_{\mathrm{pr}1} + C_{\mathrm{pr}3}\right)C_{\mathrm{pr}5}}{\omega_0^2 rC_{\mathrm{pr}1}RC_{\mathrm{pr}3}C_{\mathrm{pr}5}}\right]\left(\omega_{\mathrm{N}}^9 + \omega_{\mathrm{N}}\right) - \right.$$
$$- 4\left[C_{\mathrm{sr}2} + C_{\mathrm{sr}4} + \frac{C_{\mathrm{sr}2}C_{\mathrm{sr}4}}{4C_{\mathrm{pr}3}} + \frac{\left(4C_{\mathrm{pr}1} + C_{\mathrm{sr}2}\right)C_{\mathrm{pr}5} + C_{\mathrm{pr}1}C_{\mathrm{sr}4} + \left(4C_{\mathrm{pr}1} + C_{\mathrm{sr}2} + C_{\mathrm{sr}4} + 4C_{\mathrm{pr}5}\right)C_{\mathrm{pr}3}}{4\omega_0^2 rC_{\mathrm{pr}1}RC_{\mathrm{pr}3}C_{\mathrm{pr}5}}\right]\left(\omega_{\mathrm{N}}^7 + \omega_{\mathrm{N}}^3\right) +$$
$$\left. + 6\left[C_{\mathrm{sr}2} + C_{\mathrm{sr}4} + \frac{C_{\mathrm{sr}2}C_{\mathrm{sr}4}}{3C_{\mathrm{pr}3}} + \frac{3C_{\mathrm{pr}3} + C_{\mathrm{sr}4} + 3C_{\mathrm{pr}5}}{3\omega_0^2 rC_{\mathrm{pr}3}RC_{\mathrm{pr}5}} + \frac{C_{\mathrm{sr}2}\left(2C_{\mathrm{pr}3} + C_{\mathrm{sr}4} + 2C_{\mathrm{pr}5}\right) + 2C_{\mathrm{pr}3}\left(C_{\mathrm{sr}4} + 3C_{\mathrm{pr}5}\right)}{6\omega_0^2 rC_{\mathrm{pr}1}RC_{\mathrm{pr}3}C_{\mathrm{pr}5}}\right]\omega_{\mathrm{N}}^5\right\}^2$$

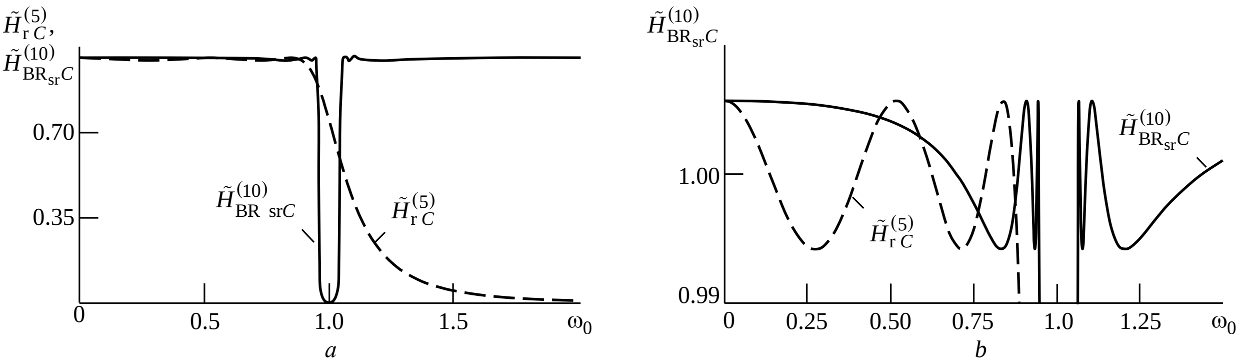

Fig. 14. Equiripple frequency responses of LPF prototype and BRF (*a*); central parts of LPF and BRF frequency responses (*b*)

Further BRF design with non-equiripple frequency response does not differ from the BPF design when appropriate replacements of analytical expressions are carried out. This allows the number of designed filter elements having values that diverse from the standard series to be reduced to zero.

Conclusion. The presented calculation methods of band filters and provided example demonstrate the feasibility and practicality of a filter design method based on solving systems of non-linear equations. In contrast to approximation approaches to determining ideal filter frequency response using special functions [14], [15] and tabular filter design [16], the presented method allows high-order filter calculation for any initial requirements without using reference data.

REFERENCES

1. Popov P. A. *Raschet chastotnykh elektricheskikh fil'trov* [Calculation of Frequency Electric Filters]. Moscow, Leningrad, *Energiya*, 1966, 216 p. (In Russ.)

2. Jonson D., Jonson J., Moore H. A handbook of active filters. New Jersey, Prentice-Hall, Inc., Englewood Cliffs, 1980, 128 p.

3. Paarmann L. D. Design and analysis of analog filters: A signal processing perspective. Dordrecht: Springer, 2014, 456 p.

4. Budak A. Passive and active network analysis and synthesis. Atlanta, London: Houghton Mifflin company, 1974, 733 p.

5. Matkhanov P. N. *Osnovy sinteza lineinykh elektricheskikh tsepei* [Basics of Linear Electrical Circuit Synthesis]. Moscow, *Vysshaya shkola*, 1978, 208 p. (In Russ.)

6. Kaufman M., Seidman A. Handbook of electronics calculation for engineers and technicians, in 2 vols. Vol. 2, McGraw-Hill, New York, 1988, 288 p.

7. Winder S. Analog and digital filter design. 2nd ed. New York: Elsevier Science, 2002, 450 p.

8. Chervinskiy E. N. Frequency Responses Resistance to Variations of Electric Filter Parameters. Journal of the Russian Universities. Radioelectronics. 2017, no. 3, pp. 24–38. (in Russ.)

9. GOST Standard 28884-90 (IEC 63-63). Preferred Number Series for Resistors and Capacitors. Moscow, Standardinform, 2006, 13 p. (in Russ.)

10. Chervinskiy E. N. Computation of Transfer Functions of Filters with Equiwave at the Section and Infinite Half-interval Amplitude-Frequency Responses. Journal of the Russian Universities. Radioelectronics. 2014, no. 4, pp. 13–28. (in Russ.)

11. Znamenskii A. E., Popov E. S. Perestraivaemye elektricheskie fil'try [Tunable Electrical Filters]. Moscow, *Svyaz*', 1979, 128 p. (In Russ.)

12. Ounrou N. E., Grigoriev E. V. Tuned quasipolinomial bandstop filters of the third order on the lumped elements. Journal of the Russian Universities. Radioelectronics. 2007, no. 6, pp. 37–45. (in Russ.)

13. Baskakova A. E., Turgaliev V. M., Kholodnyak D. V. A Tunable Lumped-Element Bandpass Filter with Independent Continuous Tuning of Centre Frequency and Bandwidth. Journal of the Russian Universities. Radioelectronics. 2016, no. 3, pp. 25–32. (in Russ.)

14. Rhodes J. D. Theory of electrical filters, Willey, London, 1977, 224 p.

15. Huelsman Lawrence P., Allen Phillip E. Introduction to the theory and design of active filters, McGraw-Hill. 1980, 384 p.

16. Saal R. Handbuch zum Filterenwuef, AEG – Telefunken, Berlin, 1979.

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СПИСОК ЛИТЕРАТУРЫ

1. Попов П. А. Расчет частотных электрических фильтров. М.–Л.: Энергия, 1966. 216 с.

2. Джонсон Д., Джонсон Дж., Мур Г., Справочник по активным фильтрам. М.: Энергоатомиздат, 1983. 128 с.

3. Paarmann L. D. Design and analysis of analog filters: A signal processing perspective. Dordrecht: Springer, 2014. 456 p.

4. Budak A. Passive and active network analysis and synthesis. Atlanta, London: Houghton Mifflin company, 1974. 733 p.

5. Матханов П. Н. Основы синтеза линейных электрических цепей. М.: Высш. шк., 1978. 208 с.

6. Кауфман М., Сидман А. Г. Практическое руководство по расчетам схем в электронике: справ.: в 2 т. Т. 2 / пер. с англ.; под ред. Ф. Н. Покровского. М.: Энергоатомиздат, 1993. 288 с.

7. Winder S. Analog and digital filter design. 2nd ed. New York: Elsevier Science, 2002. 450 p.

8. Червинский Е. Н. Устойчивость частотных характеристик к изменениям параметров электрического фильтра // Изв. вузов России. Радиоэлектроника. 2017. № 3. С. 24–38.

9. ГОСТ 28884–90 (МЭК 63-63). Межгосударственный стандарт. Ряды предпочтительных значений для резисторов и конденсаторов. М.: Стандартинформ, 2006. 13 с.

10. Червинский Е. Н. Расчет передаточных функций фильтров с равноволновыми на отрезке и бесконечном полуинтервале амплитудно-частотными характеристиками // Изв. вузов России. Радиоэлектроника. 2014. № 4. С. 13–28.

11. Знаменский А. Е., Попов Е. С. Перестраиваемые электрические фильтры. М.: Связь, 1979. 128 с.

12. Унру Н. Э., Григорьев Е. В. Перестраиваемые квазиоптимальные режекторные фильтры третьего порядка на сосредоточенных элементах // Изв. вузов России. Радиоэлектроника. 2007. № 6. С. 37–45.

13. Баскакова А. Э., Тургалиев В. М., Холодняк Д. В. Перестраиваемый полосно-пропускающий фильтр на элементах с сосредоточенными параметрами с независимым непрерывным управлением центральной частотой и шириной полосы пропускания // Изв. вузов России. Радиоэлектроника. 2016. № 3. С. 25–32.

14. Роудз Дж. Д. Теория электрических фильтров / пер. с англ.; под ред. А. М. Трахтмана. М.: Сов. радио, 1980. 240 с.

15. Хьюлсман Л. П., Аллен Ф. Е. Введение в теорию и расчет активных фильтров / пер. с англ.; под ред. А. Е. Знаменского. М.: Радио и связь, 1984. 384 с.

16. Зааль Р. Справочник по расчету фильтров / пер. с нем.; под ред. Н. Н. Слепова. М.: Радио и связь, 1983. 752 с.

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