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# Analytical Dispersion Theory for Optical Waves in Regular Microwaveguides

***Abstract.*** *A method for analysis of dispersion characteristics of guided optical modes propagating in the optical waveguides with small cross-sections is proposed. The method is based on introduction of a correction factor for a longitudinal wavenumber of propagating modes. The correction factor arises when a cross-section of the basic rectangular waveguide is subjected to perturbation. The electromagnetic field distributions along with the mode longitudinal wavenumber are found by means of variable separation method. The longitudinal wavenumber correction factor is analytically calculated in terms of coupled mode theory. The combined use of the complete set of equations of electrodynamics together with the concept of effective sources gives rise to the correction factor in the form of an intermodal coupling coefficient. It is pointed out that the coupling coefficient consists of two components, namely bulk and surface, owing to accurate account of the electrodynamics boundary conditions. Using the method proposed, the dispersion characteristics of the fundamental modes propagating in the practically employed optical waveguides having a trapezoidal cross-section are calculated. An impact of the waveguide cross-section shape to cladding dielectric constant ratio on the mode dispersion characteristics is analyzed. The necessity to take into consideration an imperfection of the waveguide cross-section in a wide range of operating wavelengths is demonstrated.*

**Keywords:** Optical Waveguides, Integral Optics, Microwave Photonics, Coupled-Mode Theory

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## Аналитическая теория дисперсии оптических волн регулярных микроволноводов

***Аннотация.*** *Разработан метод анализа дисперсионных характеристик направляемых мод в регулярных оптических микроволноводах малого поперечного сечения. Метод основан на введении поправок к продольному волновому числу мод прямоугольного волновода, выбранного в качестве базового, при искажении формы его поперечного сечения. Распределения электромагнитного поля и продольного волнового числа базового волновода рассчитываются методом разделения переменных. Поправка к продольному волновому числу рассчитывается аналитически в терминах теории связанных мод. Указанная поправка в виде коэффициента межмодовой связи возникает на основании совместного использования полной системы уравнений Максвелла при введении понятия об эффективных источниках. Показано, что последовательный учет граничных условий электродинамики приводит к форме коэффициента связи, включающей объемную и поверхностную составляющие. Разработанный метод применен для расчета дисперсионных характеристик низших волноводных мод, распространяющихся в микроволноводах трапециевидного сечения, применяемых на практике. Продемонстрировано влияние поперечного сечения микроволновода на дисперсионные характеристики мод в зависимости от соотношения сторон, а также от отношения значений диэлектрических проницаемостей сердцевины микроволновода и его оболочки. Показана необходимость учета влияния формы микроволновода на дисперсионные характеристики мод в широком диапазоне значений рабочих длин волн и при различных распределениях диэлектрической проницаемости волноведущей структуры.*

**Ключевые слова:** оптические волноводы, интегральная оптика, радиофотоника, теория связанных мод

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**Introduction.** Within the last two decades, the field of microwave photonics (MWP) has been rapidly developing [1]–[3]. At the same time, a comparatively independent research area has developed as a part of the field. It was named as integrated microwave photonics (IMWP) [4], [5]. One of the IMWP key elements is a thin-film dielectric optical waveguide as well as components built from such waveguides [6], [7]. It should be noted that specific nature of the planar technology used to produce optical waveguides results in deviation of their cross-section from a rectangular shape [7], [8]. The non-rectangular shape of the waveguide cross-section affects dispersion characteristics of propagating modes and demands extending already existing theories for wave properties of optical waveguides.

According to literature, there are several techniques to be used for calculation of dispersion characteristics of modes in optical waveguides with an arbitrary cross-section. They include a circular harmonics method based on a waveguide field expansion into an infinite series of Bessel and Hankel functions [8], [9], a method combining a series expansion and a contour integration [10], a perturbation theory method [11] as well as the coupled-mode theory method [12]. Note that methods [8]–[10] are rather cumbersome and compute-intensive. Therefore their practical application imposes the use of certain assumptions [13]. Such assumptions due to commensurability of a waveguide cross-section with operating wavelengths may have an uncontrollable impact on dispersion characteristics of propagating waves.

In addition to analytical ones, other methods of simulation of trapezoidal cross-section optical waveguides are developed. They include e.g. a finite difference method [14], an equivalent circuit method [15], etc.

Among the forenamed calculation methods for mode dispersion the special mention should go to the method based on the use of the complete system of equations of electrodynamics and the coupled-modes theory in combination with the concept of "effective sources" [12]. This method allows for analytical description of the waveguide dispersion properties with arbitrary behavior ("modulation") of their cross-section.

The goal of this article is to develop an analytical theory enabling to precisely describe dispersion characteristics of guided optical waves propagating in regular dielectric microwaveguides of non-rectangular cross-section.

**Dispersion characteristics of modes of a rectangular dielectric waveguide.** First, we turn our attention to analysis of the dispersion characteristics

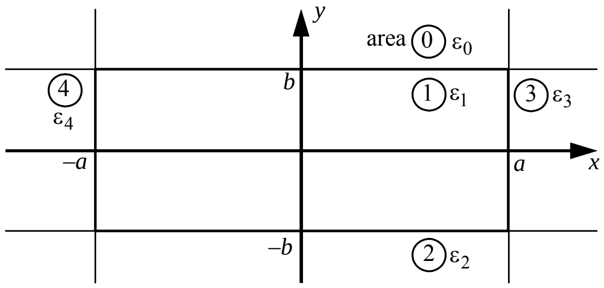


*Fig. 1*

of guided modes in the lossless dielectric waveguide with a rectangular cross-section because such waveguide is chosen as a reference one in handling our problem. Such a dielectric structure containing a rectangular waveguide is shown in fig. 1. The waveguide core has width $2a$ and height $2b$, a dielectric permittivity of $\varepsilon_1$, and it is surrounded by dielectrics with the permittivities $\varepsilon_0$, $\varepsilon_2$, $\varepsilon_3$ и $\varepsilon_4$. To calculate mode dispersion characteristics, we use the method of approximate analysis which basically is a method of separation of variables [13].

Note that in solving of the boundary value problem four cases can be distinguished [16], [17] that correspond to different combinations of trigonometric functions. Each combination describes a set of propagating Eigen modes which together form the infinite set of modes. The following derivation concerns the lowest-type guided modes of two polarizations, namely $E_x^{11}$ and $E_y^{11}$. The expressions for the other modes are not given due to their analogy.

For the chosen modes the fields at frequency $\omega$ within the waveguide core (region 1, $-a < x < a$ and $-b < y < b$) have the following form:

$$E_{1z} = E_1^{\mathrm{m}} \begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\left(xk_{1x}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\left(yk_{1y}\right) e^{-i(\beta z-\omega t)};$$
$$H_{1z} = H_1^{\mathrm{m}} \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\left(xk_{1x}\right) \begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\left(yk_{1y}\right) e^{-i(\beta z-\omega t)}, \quad (1)$$

where $E_1^{\mathrm{m}}$ and $H_1^{\mathrm{m}}$ are constants meaning amplitude; $k_{1x}$ and $k_{1y}$ are transverse wave numbers within the waveguide; $\beta$ is an unknown longitudinal wave number. In the expression (1) the upper trigonometric functions describe the waveguide mode $E_x^{11}$, and the lower ones describe $E_y^{11}$ mode. Hereinafter for the sake of simplicity the time factor $\exp(-i\omega t)$ is omitted and the cross-sectional field distributions are marked with circumflex.

Outside the waveguide in the regions 3 (where $x \ge a$ and $0 < y < b$) and 0 (where $y \ge b$ and $0 < x < a$) the expressions for electric field take the form of

$$\hat{E}_{3z} = \hat{E}_{1z}(a, y)\exp\left[-(x-a)k_{3x}\right];$$
$$\hat{E}_{0z} = \hat{E}_{1z}(x, b)\exp\left[-(y-b)k_{0y}\right],$$

where $k_{3x}$ and $k_{0y}$ are the components of the outside transverse wavenumber. Their corresponding expressions read:

$$k_{0y} = \sqrt{\omega^2(\varepsilon_1 - \varepsilon_0)\mu_0 - k_{1y}^2};$$
$$k_{3x} = \sqrt{\omega^2(\varepsilon_1 - \varepsilon_3)\mu_0 - k_{1x}^2},$$

where $\mu_0$ stands for the vacuum permeability. In the corner regions of $x \ge a$ and $y \ge b$, the fields symmetrical against the 0*x* and 0*y* axes are considered equal to zero.

Consider next the case of $\varepsilon_0 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 \equiv \equiv \varepsilon_2$, that will make possible to derive a dispersion equation by imposing the electrodynamics boundary conditions only along $x = a$ and $y = b$ waveguide walls.

The transverse field components in its turn are expressed by means of the longitudinal ones derived from Maxwell's equations. Imposition of the continuity boundary conditions of electrodynamics on the transverse field components produces a set of equations for the components of the transverse wavenumber of the modes:

$$\begin{aligned} &k_{1x}k_{3x}\,{}^{\mathrm{ctg}}_{\mathrm{tg}}(ak_{1x}) - k_{1y}k_{2y}\,{}^{\mathrm{tg}}_{\mathrm{ctg}}(bk_{1y}) \pm k_{t2}^2 = 0;\\ &k_{1x}k_{3x}\,{}^{\mathrm{tg}}_{\mathrm{ctg}}(ak_{1x}) - k_{1y}k_{2y}\,{}^{\mathrm{tg}}_{\mathrm{ctg}}(bk_{1y}) \mp \varepsilon_r k_{t2}^2 = 0, \end{aligned} \qquad (2)$$

where $\varepsilon_r = \varepsilon_1/\varepsilon_2$, and the outside transverse wavenumber $k_{t2}$, as indicated by the "t" subscript. It can be expressed as follows:

$$k_{t2}^2 = k_0^2(\varepsilon_1 - \varepsilon_2) - k_{1x}^2 - k_{1y}^2.$$

In the set of equations (2) the upper line corresponds to the mode $E_x^{11}$, and the lower one to the mode $E_y^{11}$. From the set of equations (2) we find the components of the transverse wavenumber $k_{1x}$ and $k_{1y}$, which occur in the expression for the propagation constant

$$\beta^2 = k_1^2 - k_{1x}^2 - k_{1y}^2, \qquad (3)$$

where $k_1^2$ is the square absolute value of the inside wave vector, that is equal to $k_1^2 = k_0^2\varepsilon_1 = \omega^2\varepsilon_1\mu_0$.

**Introduction of effective sources.** Now we turn to finding dispersion characteristics of the trapezoidal waveguide. To do this we employ coupled-mode theory [11]. Following the theory let us write electric and magnetic fields as an expansion in Eigen modes of the rectangular waveguide:

$$\mathbf{E} = \sum_n A_n \hat{\mathbf{E}}_n e^{-i\beta_n z};$$
$$\mathbf{H} = \sum_n A_n \hat{\mathbf{H}}_n e^{-i\beta_n z}, \qquad (4)$$

where $A_n$ are the mode excitation amplitudes; $\hat{\mathbf{E}}_n$ and $\hat{\mathbf{H}}_n$ are the waveguide modes derived from the solutions of Maxwell's equations in the section above; $\beta_n$ is a mode propagation constant.

In the expansion (4) the radiative modes are not explicitly emphasized. However, they can be taken into account if we consider summation signs in generalized sense, including integrating on continuous argument.

Availability of the excitation regions in the waveguiding structure changes dispersion characteristics of an ideal waveguide. Note that excitation can result from both availability of the real electromagnetic field source and changes in the environment parameters. Mathematically both excitation types are described by means of the excitation currents which are a part of Maxwell's equations. Everywhere outside excitation areas the field is described as a sum of Eigen functions (4).

Let us write the expressions for the fields inside the excitation areas. To do this, let us make use of the coupled-mode theory inherent assumption of that the expansion amplitudes $A_n$ acquire longitudinal dependence in the excitation area. Moreover, as you can see in the monographs [18], [19], in excitation area the expansions (4) lose their force. Thus, they need to include longitudinal fields:

$$\mathbf{E} = \sum_n A_n(z)\hat{\mathbf{E}}_n e^{-i\beta_n z} + \mathbf{E}_b;$$
$$\mathbf{H} = \sum_n A_n(z)\hat{\mathbf{H}}_n e^{-i\beta_n z} + \mathbf{H}_b, \qquad (5)$$

where $\mathbf{E}_b$ and $\mathbf{H}_b$ are called orthogonal complementary fields. They represent orthogonal complement to Hilbert space spanned on the waveguide basis functions. In (5) the "b" subscript indicates the bulk nature of the fields.

Now following the coupled-mode theory, we introduce the effective sources of excitation. In isotropic medium, scalar dielectric and magnetic permeability occur in the cons constitutive equations as follows:

$$\mathbf{D} = \varepsilon\mathbf{E};\ \mathbf{B} = \mu\mathbf{H}.$$

Excitation of the medium changes field distributions introducing the excessive inductions

$$\Delta\mathbf{D} = \Delta\varepsilon\mathbf{E};\ \Delta\mathbf{B} = \Delta\mu\mathbf{H},$$

where $\Delta\varepsilon = \varepsilon_{\text{pert}}(\mathbf{r}_t, z) - \varepsilon(\mathbf{r}_t)$; $\Delta\mu = \mu_{\text{pert}}(\mathbf{r}_t, z) - \mu(\mathbf{r}_t)$; $\mathbf{r}_t$ is a coordinate in the transverse plane; $\varepsilon_{\text{pert}}(\mathbf{r}_t, z)$ and $\mu_{\text{pert}}(\mathbf{r}_t, z)$ are the complete permittivity and permeability of perturbed medium; while $\varepsilon(\mathbf{r}_t)$ and $\mu(\mathbf{r}_t)$ are the permittivity and permeability of nonperturbed medium. They are independent of time because they are purely geometrical in nature. At optical frequencies $\mu_{\text{pert}}(\mathbf{r}_t, z) = \mu(\mathbf{r}_t) = 1$, which gives $\Delta\mu = 0$. Hence, in the considered particular case the excessive induction $\Delta\mathbf{B} = 0$. We mention in passing that introduction of excessive inductions is similar to "polarization perturbation" described in [20]. The excessive inductions produce the excessive bias currents

$$\mathbf{J}_b^e = i\omega\Delta\mathbf{D};\quad \mathbf{J}_b^m = i\omega\Delta\mathbf{B} = 0, \tag{6}$$

where the "e" and "m" superscripts emphasize either electric or magnetic nature of the corresponding current.

Now we obtain an expression for effective bulk electric current. With regard to (6) we now write down Maxwell's equation as

$$\begin{aligned} \nabla \times \mathbf{E} &= -i\omega\mathbf{B}, \\ \nabla \times \mathbf{H} &= i\omega\mathbf{D} + \mathbf{J}_b^e. \end{aligned} \tag{7}$$

Substitution of **Ошибка! Источник ссылки не найден.** in (7) allows to express the effective bulk electric current in terms of orthogonal complementary fields:

$$E_b = -\frac{1}{i\omega\varepsilon} J_{bz}^e,\quad H_b = 0.$$

Next, we show that besides bulk currents, the effective sources of excitation are to include surface currents as well. For this purpose, we write down boundary conditions in a common form on a contour $L$ that encloses cross-section of the excitation area $S$ (fig. 2):

$$\begin{aligned} \mathbf{n}^+ \times \mathbf{E}^+ + \mathbf{n}^- \times \mathbf{E}^- &= -\mathbf{J}_s^m; \\ \mathbf{n}^+ \times \mathbf{H}^+ + \mathbf{n}^- \times \mathbf{H}^- &= \mathbf{J}_s^e, \end{aligned} \tag{8}$$

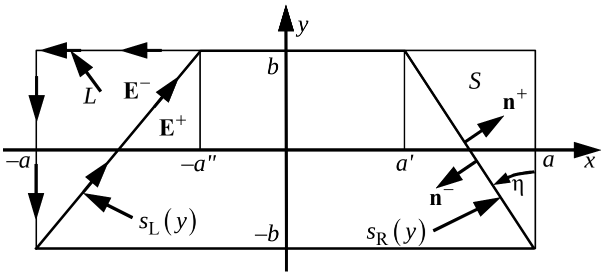


*Fig. 2*

where $\mathbf{E}^+$ and $\mathbf{E}^-$ designate the electric field inside and outside of $S$; $\mathbf{H}^+$ and $\mathbf{H}^-$ designate the magnetic field inside and outside of $S$. The normals $\mathbf{n}^+$ and $\mathbf{n}^-$ are directed inside and outside of $S$. The "s" subscript underlines the surface nature of the currents. The designations introduced in (8) are represented on fig. 2.

The expressions (8) stem from the fact that the fields inside the region $S$ (5) differ from the fields outside $S$ (4) by the value of $\mathbf{E}_b$. Substitution of decompositions (4) and (5) in the conditions (8) makes it possible to obtain an expression for effective surface currents that are written as:

$$\mathbf{J}_s^e = -\frac{\mathbf{e}_z \times \mathbf{n}}{i\omega\varepsilon} J_{bz}^m\Big|_L;\quad \mathbf{J}_s^m = 0.$$

Here $\mathbf{e}_z$ is the longitudinal unit vector and $\mathbf{n}$ is the outside-pointing normal to $S$.

**Correction for longitudinal wave number.** For the purpose of derivation of the set of coupled mode equations for the perturbed system it is necessary to obtain an expansion of the effective sources in terms of the Eigen modes of the unperturbed system. Next, the obtained expression should be substituted in Lorentz lemma written in conjugate form. Hence, it is possible to find an expression describing excitation of the $m$-th waveguide mode with the set of all Eigen modes of the waveguiding structure

$$da_m(z)/dz = -i\beta_m a_m(z) + \sum_n \kappa_{mn} a_n(z), \tag{9}$$

where $a_n(z) = A_n(z)\exp(-i\beta z)$. Note that in (9) the coupling factor is introduced, consisting of two parts. The first one is produced by the bulk and the second one by the surface excitation source. It has the following form

$$\kappa_{mn} = \kappa_{mn}^b + \kappa_{mn}^s,$$

where the corresponding "b" and "s" subscripts carry the same meaning as in the previous section but were moved upwards for further notational convenience. The first one is induced by the bulk sources of excitation and the second one by the surface ones. As calculations show, the expressions for the bulk and surface coupling factors look like this:

$$\kappa_{mn}^{\mathrm{b}} = -\frac{i\omega}{N_m}\int_S \left(\Delta\bar{\varepsilon}\,\hat{\mathbf{E}}_n\right)\hat{\mathbf{E}}_m^{*}\,dS;$$
$$\kappa_{mn}^{\mathrm{s}} = -\frac{1}{N_m}\int_L \left(\Delta\bar{\xi}\,\hat{\mathbf{E}}_n\right)\hat{\mathbf{H}}_m^{*}\,dL, \qquad (10)$$

where $N_m$ is a normalizing factor; $\Delta\bar{\varepsilon}$ and $\Delta\bar{\xi}$ are the tensors of static surface coupling which describe geometrical perturbation of the waveguide. Their use makes it possible to considerably simplify writing the expressions for mode decomposition of the effective sources. Normalizing factor $N_m$ is related to the mode power flow density:

$$N_m = 2\,\mathrm{Re}\int_C \left[\hat{\mathbf{E}}_m^{*}\times\hat{\mathbf{H}}_m\right]\mathbf{e}_z\,dC,$$

where $C$ is the contour encloses the waveguide and its surroundings.

Note that the coupling factor obtained here differs from conventionally used [21] by the element $\kappa_{mn}^{\mathrm{s}}$. The element occurrence in the intermode coupling is caused by introduction of the effective sources and their description in terms of orthogonal complementary fields.

We emphasize that the expression (10) enables considering waveguides with perturbations of different nature. As an example, we mention periodical modulation of the waveguide cross-section and/or periodical modulation of the dielectric permittivity of the waveguide material, the waveguide bends, etc.

Below we consider a particular case of lowest-type propagating mode in a regular dielectric waveguide having nonrectangular cross-section. A distinctive feature of such mode is lack of interaction with other modes. The propagation constant of a regular nonrectangular waveguide $\beta_m'$ is related to the propagation constant of the reference rectangular waveguide $\beta_m$ by means of the coupling factor:

$$\beta_m' = \beta_m + i\kappa_{mm}.$$

This last expression can be derived from (10) by taking element $\kappa_{mm}a_m(z)$ out of summation symbol. The coupling tensors defining $\kappa_{mm}$ in the case of trapezoidal cross-section waveguide take the form of

$$\Delta\bar{\varepsilon}(y) = \Delta\varepsilon(y)\left[\bar{\mathbf{I}} - \mathbf{e}_z\mathbf{e}_z\frac{\varepsilon_1(y)}{\varepsilon_2(y)}\right];$$
$$\Delta\bar{\xi}(y) = \mathbf{e}_{\mathrm{t}}\mathbf{e}_z\left.\frac{\left[\varepsilon_2(y) - \varepsilon_1(y)\right]}{\varepsilon_2(y)}\right|_L, \qquad (11)$$

where $\bar{\mathbf{I}}$ is a unity matrix; $\mathbf{e}_{\mathrm{t}}$ is a unit vector tangent to contour $L$, $\mathbf{e}_z\mathbf{e}_z$ is a dyad.

To define the integration limits in (10) we describe the lateral side of trapezoidal cross-section by means of function $s_{\mathrm{L}}(y)$ or $s_{\mathrm{R}}(y)$ (see fig. 2). In this case, with regard to the expressions (11), the integrals (10) assume the following form:

$$\kappa_{mn}^{\mathrm{b}} = -\frac{i\omega}{N_m}\int_{-b}^{b}\int_{-a}^{-a+s_{\mathrm{L}}(y)}\hat{\Psi}\,dxdy + \frac{i\omega}{N_m}\int_{-b}^{b}\int_{a}^{a-s_{\mathrm{R}}(y)}\hat{\Psi}\,dxdy;$$

$$\kappa_{mn}^{\mathrm{s}} = -\frac{1}{N_m}\int_{-b}^{b}\int_{-a}^{-a+s_{\mathrm{L}}(y)}\hat{\Xi}\,dxdy - \frac{1}{N_m}\int_{-b}^{b}\int_{a}^{a-s_{\mathrm{R}}(y)}\hat{\Xi}\,dxdy,$$

where

$$\hat{\Psi} = \left(\varepsilon_2 - \varepsilon_1\right)\left(\hat{\mathbf{E}}_{mt}^{*}\hat{\mathbf{E}}_{nt} + \frac{\varepsilon_1}{\varepsilon_2}\hat{E}_{mz}^{*}\hat{E}_{nz}\right);$$
$$\hat{\Xi} = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2}\left[\frac{\partial}{\partial x}\left(\hat{H}_{my}^{*}\hat{E}_{nz}\right) - \frac{\partial}{\partial y}\left(\hat{H}_{mx}^{*}\hat{E}_{nz}\right)\right].$$

In the last expression the notation $\hat{\mathbf{E}}_{nt}$ is introduced based on $\hat{\mathbf{E}}_n = \hat{\mathbf{E}}_{nt} + \mathbf{e}_z\hat{E}_{nz}$.

**Simulation results.** Following the above described analytical theory we perform analysis of the influence of width ratio $a'/a$ to the dispersion characteristics of microwaveguides. In doing so, we specify that $a'' = a'$ (see fig. 2). The introduced $a'/a$ factor can vary between 0 and 1, which corresponds to changing of the cross-section shape from triangle to rectangular. It is also convenient to introduce deviation angle η of trapezoidal waveguide lateral wall from the reference rectangular one. To demonstrate the simulation results we employ the normalized coordinates:

$$B^2 = \frac{\beta^2/k_0^2 - \varepsilon_2}{\varepsilon_1 - \varepsilon_2};\; V = k_0\frac{2b}{\pi}\sqrt{\varepsilon_1 - \varepsilon_2}.$$

Fig. 3 presents the results of theoretical modeling of $E_x^{11}$-mode dispersion characteristic of optical microwaveguide obtained for different values of $a'/a$ factor and η angle. The microwaveguide under consideration has the cross-section dimensions $a = 1.4$ μm and

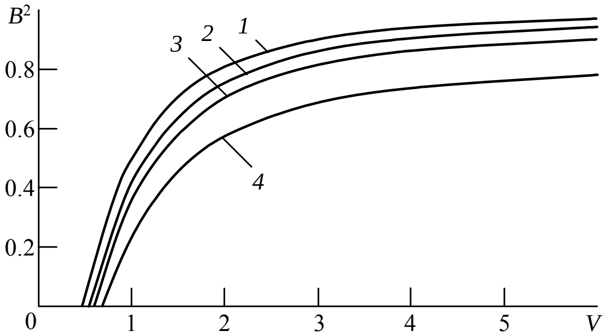


*Fig. 3*

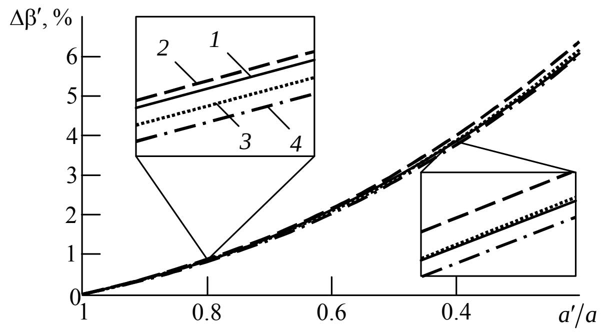


*Fig. 4*

$b = 0.7$ μm and permittivity $\varepsilon_1 = 1.98$ and $\varepsilon_2 = 1.44$. The digits on curves designate the following: *1* – $a'/a = 1.0$; *2* – $a'/a = 0.58$; *3* – $a'/a = 0.4$; *4* – $a'/a = 0.1$. From fig. 3 it follows that for the same wavelength the propagation factor decreases with increasing the sidewall angle. From the physical point of view, this is caused by increase of the transverse wave number and is in agreement with the general formula (3).

Fig. 4 shows simulated dependencies of the propagation factor deviation on $a'/a$ ratio for the waveguides with different aspect ratios. The value plotted on the 0*x*-axis is [%]

$$\Delta\beta' = \frac{\beta_m - \beta_m'}{\beta_m} 100,$$

with $\beta_m$ values taken at the wavelength of 1.55 μm, which is typical for optical *C*-band. The digits on curves designate the following: *1* – $a/b = 4$; *2* – $a/b = 3$; *3* – $a/b = 2$; *4* – $a/b = 1$. The dielectric permittivities employed in the simulation correspond to the previous case. The behavior of the curves in fig. 4 are nearly identical. This points to the fact that there is no "preferable" cross-section aspect ratio for minimization of trapezoidal shape impact on the waveguide dispersion characteristics.

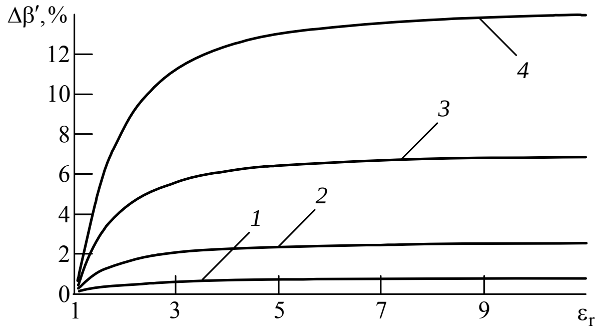


*Fig. 5*

Fig. 5 shows a set of the curves that represent the propagation factor dependency on the core-to-surroundings relative permittivity $\varepsilon_r$ for different values of the η angle. The microwaveguide under consideration has the cross-section dimensions $a = 1.4$ μm and $b = 0.7$ μm. The digits on curves designate the following: *1* – $a'/a = 0.9$; *2* – $a'/a = 0.7$; *3* – $a'/a = 0.4$; *4* – $a'/a = 0.1$. The parameter $\varepsilon_r$, which expresses the ratio of the dielectric constant of the core of the microwaveguide and the surrounding space, determines the degree of concentration of the mode field in the core of the waveguide. For a small value of $\varepsilon_r$ the perturbation of the cross-section of the waveguide has a weak effect on the dispersion characteristics of the modes, since the field of the main mode is concentrated in the surrounding space. Thus, based on the data presented, it should be concluded that for the waveguides with a high $\varepsilon_r$ value, it is especially important to take into account the effect of the non-rectangular shape of the cross section on the dispersion characteristics of the modes.

In conclusion, this paper offers an analytical theory for the dispersion characteristics of the guided modes propagating in the regular optical microwaveguides with small cross-sections. The theory relies on the calculation of the corrections to the propagation factor by means of the coupled mode theory with introduction of the effective excitation sources. Based on the developed theory, the dispersion characteristics of the guided modes in the optical dielectric waveguides with the trapezoidal cross-section are calculated. The microwaveguide cross-section shape impact on the dispersion characteristics as a function of the waveguide aspect ratio, as well as the ratio of the dielectric permittivities of the microwaveguide and the surrounding space are revealed.

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